Roll No. ....

D–3764

M. A./M. Sc. (Final) EXAMINATION, 2020

MATHEMATICS

(Optional)

Paper Fifth (*iii*)

(Fuzzy Sets and Their Applications)

Time : Three Hours] [*Maximum Marks : 100*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define fuzzy set with examples. Let A, B be fuzzy sets defined on a universal set X. Prove that :

$$|A| + |B| = |A \cup B| + |A \cap B|$$

(b) Let f be a decreasing generator. Then a function g defined by $g(a) = f(0) - f(a)$ for any $a \in [0,1]$ is an increasing generator with $g(1) = f(0)$ and its pseudo-inverse $g^{(-1)}$ is given by :

$$g^{(-1)}(a) = f^{(-1)}(f(0) - a)$$

for any $a \in R$ and all find the value of $g_w^{(-1)}(a)$ if $g_w(a) = (1-a)^w$ $(w > 0)$.

(c) Let X be any universal set. A be a fuzzy set defined on X, then prove that for every $A \in \mathbf{F}(X)$ (family of set X)

$$A = \bigcup_{\alpha \in [0,1]} {}_{\alpha}A$$

where :

$${}_{\alpha}A(x) = \alpha . {}^{\alpha}A(x) \; \forall x \in X$$

and union is standard fuzzy union. Also verify this if :

$$A = \frac{.2}{x_1} + \frac{.4}{x_2} + \frac{.6}{x_3} + \frac{.8}{x_4} + \frac{1}{x_5}$$

Unit—II

2. (a) Let $* \in \{+, -, ., /\}$ and let A, B denote continuous fuzzy numbers. Then prove that A * B is continuous fuzzy number, where :

$$(A * B)(z) = \sup_{z = x * y} \min[A(x), B(y)]$$

(b) Consider the set :

$$X_1 = \{0, 1\}$$

$$X_2 = \{0, 1\}$$

$$X_3 = \{0, 1, 2\}$$

and the ternary fuzzy relation on $X_1 \times X_2 \times X_3$ defined in table ahead. Let $R_{ij} = [R \downarrow \{X_i, X_j\}]$ and $R_i = [R \downarrow \{x_i\}]$ for all $i, j \in \{1, 2, 3\}$. Compute the projection R_{12}, R_{23} and R_{13}. Also find cylindric closures of (R_{12}, R_{13}, R_{23}) :

(x_1, x_2, x_3)	$R(x_1, x_2, x_3)$
0 0 0	0.4
0 0 1	0.9
0 0 2	0.2
0 1 0	1.0
0 1 1	0.0
0 1 2	0.8
1 0 0	0.5
1 0 1	0.3
1 0 2	0.1
1 1 0	0.0
1 1 1	0.5
1 1 2	1.0

(c) Explain fuzzy equivalence relations and fuzzy compatibility relation.

Unit—III

3. (a) Determine all solution of $p \circ Q = r$, where :

$$p = [p_j \;/\; j \in J]$$

$$Q = [q_{jk} \;/\; j \in J, k \in K]$$

$$r = [r_k \;/\; k \in K]$$

and given that :

$$Q = \begin{bmatrix} .1 & .4 & .5 & .1 \\ .9 & .7 & .2 & 0 \\ .8 & 1 & .5 & 0 \\ .1 & .3 & .6 & 0 \end{bmatrix}$$

and $r = [.8 \quad .7 \quad .5 \quad 0]$

(b) Explain evidence theory.

(c) A belief measure Bel on a finite power set P (X) is a probability measure if and only if the associated basic probability assignment function m is given by :

$$m(\{x\}) = \text{Bel}(\{x\})$$

and $m(A) = 0$

for all subsets of X are not singletons.

Unit—IV

4. (a) Write the difference between fuzzy logic and classical logic.

(b) Explain fuzzy quantifiers.

(c) Consider the if... then rules :

(i) If x is A_1, then y is B_1

(ii) If x is A_2, then y is B_2

where $A_j \in \mathbf{J}(x)$, $B_i \in \mathbf{J}(y)$ $(j = 1, 2)$ are fuzzy sets :

$$A_1 = \frac{1}{x_1} + \frac{.9}{x_2} + \frac{.1}{x_3}$$

$$A_2 = \frac{.9}{x_1} + \frac{1}{x_2} + \frac{.2}{x_3}$$

$$B_1 = \frac{1}{y_1} + \frac{.2}{y_2}$$

$$B_2 = \frac{.2}{y_1} + \frac{.9}{y_2}$$

given the fact "x is A'" where $A' = \frac{.8}{x_1} + \frac{.9}{x_2} + \frac{.1}{x_3}$.

Use the method of interpolation to calculate the conclusion B'.

Unit—V

5. (a) Formulate reasonable fuzzy inference rules for air conditioning fuzzy control system.

(b) Assume that each individual of a group of eight decision makers has a total preference ordering $P_i\ (i \in N_8)$ on a set of alternatives $X = \{w, x, y, z\}$ as follows :

$$P_1 = (w, x, y, z)$$

$$P_2 = P_5 = (z, y, x, w)$$

$$P_3 = P_7 = (x, w, y, z)$$

$$P_4 = P_8 = (w, z, x, y)$$

$$P_6 = (z, w, x, y)$$

Use fuzzy multiperson decision-making to determine the group decision.

(c) Explain Fuzzy Ranking methods.