Roll No. ....

E–763

M. A./M. Sc. (Third Semester)

EXAMINATION, Dec.-Jan., 2020-21

MATHEMATICS

Paper Second

(Partial Differential Equations in Mechanics—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 2 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. In a dynamical system of N particles with K constraint equations, the degree of freedom is :

   (a) 3N + K

   (b) 3N – K

   (c) 2N – K

   (d) 2N + K

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2. If any conjugate momentum is constant, then :

(a) $\frac{\partial H}{\partial \dot{q}_i} = 0$

(b) $\frac{\partial H}{\partial q_i} = 0$

(c) $\frac{\partial H}{\partial \dot{p}_i} = 0$

(d) $\frac{\partial H}{\partial p_i} = 0$

3. For the Hamiltonian $H = \frac{1}{2}(p^2 + q^2)$, $[\dot{p}, H]$ is :

(a) q

(b) $-q$

(c) p

(d) $-p$

4. The PDE $u_t(x,t) - u_x(x,t) = a(x-t)$ represents :

(a) Laplace's equation

(b) Wave equation

(c) Transport equation

(d) Heat equation

5. The general solutions of the PDE $u_{xy} = 0$ is $u(x, y) =$

(a) $F(x) + G(y)$

(b) $F(x).G(y)$

(c) $F(x.y)$

(d) None of the above

6. $V \in C^2(\overline{U})$ is subharmonic in open set U if :

(a) $-\Delta V \geq 0$

(b) $-\Delta V \leq 0$

(c) $DV \geq 0$

(d) $DV \leq 0$

7. The solution for membrane in wave equation is known as :

(a) Kirchhoff's formula

(b) Poisson's formula

(c) D'Alembert's formula

(d) Cauchy's formula

8. The integral of fundamental solution of heat equation is :

(a) 0

(b) –1

(c) 1

(d) ∞

9. A non-negative harmonic function in R^n is a constant, is statement of :

(a) Liouville's theorem

(b) Euler's theorem

(c) Dirichlet's principle

(d) Harnack's first theorem

10. Attraction of a thin uniform spherical shell with mass M and radius a at point on its surface is :

   (a) 0

   (b) $\gamma \frac{M}{a^2}$

   (c) $\frac{\gamma M}{4a^2}$

   (d) $\frac{\gamma M}{2a^2}$

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. State mean value formula for Laplace's equation.
2. Define harmonic function.
3. Define Green function for any region.
4. Define Hamilton's variables.
5. Define cyclic coordinates.
6. Write D'Alembert's principle.
7. Define heat ball.
8. Define potential of an attracting mass.

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Derive fundamental solution of Laplace's equation.
2. Derive uniqueness for wave equation.
3. State and prove strong maximum principle for the Laplace's equation.

4. A particle of mass moves on a smooth plane, find Lagrange's equation.
5. Write fundamental Poisson bracket.
6. Calculate Poisson bracket of P with H, where H = T + V.
7. Derive Hamiltonian as a constant of motion.
8. Derive potential of a finite rod.

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. State and prove estimates on derivatives of Laplace's equation.

   Or

   State and prove representation formula using Green's function.

2. Derive the fundamental solution of heat equation.

   Or

   State and prove Euler-Poisson-Darboux's equation.

3. State and prove the Donkin's theorem.

   Or

   Derive Routh's equations of motion.

4. Find the attraction of a uniform solid sphere at an external or internal point.

   Or

   State and prove Laplace's equation with the help of Gauss theorem.