Roll No. ....

E–765

M. A./M. Sc. (Third Semester) EXAMINATION, Dec.-Jan., 2020-21

MATHEMATICS

Paper Third (C)

(Fuzzy Set and Their Application)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. Which one of the following is triangular membership function ?

(a) $$A(x) = \begin{cases} 0 & x \le 20, x \ge 60 \\ \dfrac{x-20}{15} & 20 < x < 35 \\ \dfrac{60-x}{15} & 45 < x < 60 \\ 1 & 35 \le x \le 45 \end{cases}$$

(b) $A(x) = \begin{cases} \dfrac{x-6}{2} & 6 \le x \le 8 \\ \dfrac{10-x}{2} & 8 < x \le 10 \\ 0 & \text{otherwise} \end{cases}$

(c) Both (a) and (b)

(d) None of the above

2. If :

$$A = \frac{0.2}{x_1} + \frac{0.5}{x_2} + \frac{0.6}{x_3}$$

and $$B = \frac{0.1}{x_1} + \frac{0.4}{x_2} + \frac{0.5}{x_3}$$

then A ~ B is :

(a) $\dfrac{0.2}{x_1} + \dfrac{0.5}{x_2} + \dfrac{0.5}{x_3}$

(b) $\dfrac{0.1}{x_1} + \dfrac{0.4}{x_2} + \dfrac{0.5}{x_3}$

(c) $\dfrac{0.8}{x_1} + \dfrac{0.5}{x_2} + \dfrac{0.4}{x_3}$

(d) None of the above

3. Which one of the following properties is not true in fuzzy set ?

(a) $(A \cap B)^C = A^C \cup B^C$

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(c) $A \cup \phi = A, A \cap X = A$

(d) $A \cup A^C = X, A \cap A^C = \phi$

4. The algebraic sum of *t*-conorm is defined by :

(a) $u(a, b) = \max(a, b)$

(b) $u(a, b) = a + b - ab$

(c) $u(a, b) = \min(1, a + b)$

(d) $u(a, b) = \begin{cases} a & \text{when } b = 0 \\ b & \text{when } a = 0 \\ 1 & \text{otherwise} \end{cases}$

5. The threshold type complements is defined by :

(a) $C(a) = 1 - a$

(b) $C(a) = \dfrac{1-a}{1+\lambda a}$

(c) $C(a) = \begin{cases} 1 & \text{for } a \le t \\ 0 & \text{for } a > t \end{cases}$

(d) None of the above

6. u_{max}, i_{min} and $C(a) = 1 - a$ operation on fuzzy sets satisfies :

(a) De Morgan law

(b) Law of excluded middle

(c) Law of contradiction

(d) All of the above

7. Fuzzy set D defined on Y = {0, 1, 4, 9, ..., 100} by :

$$D = \frac{.5}{4} + \frac{.6}{16} + \frac{.7}{25} + \frac{1}{100}$$

where $f : X \to Y$ defined as $f(x) = x^2$ and X = {0, 1, 2, ..., 10}. The value of $f^{-1}(D)$ if $x = 4$:

(a) 0.5

(b) 0.6

(c) 0.7

(d) 1

8. Which one of the following conditions is correct for a fuzzy number A ?

(a) A must be a normal fuzzy set.

(b) ${}^{\alpha}A$ must be a closed interval.

(c) The support of A i.e. ${}^{0+}A$ must be bounded.

(d) All of the above

9. Let A and B be two closed intervals such that $A = [a_1, a_2]$, $B = [b_1, b_2]$. Then B – A is :

(a) $[b_2 - a_2, b_1 - a_1]$

(b) $[b_1 - a_1, b_2 - a_2]$

(c) $[b_1 - a_2, b_2 - a_2]$

(d) None of the above

10. Let R (*x*, *y*) is a fuzzy relation defined on the fuzzy sets X and Y. Then height of the fuzzy relation R (*x*, *y*) is :

(a) $\max_{y \in Y} \max_{x \in Y} R(x, y)$

(b) $\min_{y \in Y} \min_{x \in Y} R(x, y)$

(c) $\max_{y \in Y} \min_{x \in Y} R(x, y)$

(d) None of the above

11. Let R be a fuzzy partial ordering defined on set X. An element $x \in X$ is called undominated iff :

(a) $R(y, x) = 0 \ \forall y \in X$ and $y \neq x$

(b) $R(x, y) = 0 \ \forall y \in X$ and $y \neq x$

(c) $R(y, x) = 1 \ \forall y \in X$ and $y \neq x$

(d) None of the above

12. The fuzzy relation R (x, y) is called tolerance or compatibility if :

(a) R is reflexive, symmetric and transitive.

(b) R is symmetric and transitive.

(c) R is reflexive and transitive.

(d) R is reflexive and symmetric.

13. The relational join P * Q corresponding to the standard max min composition is a ternary relation $R(x, y, z)$ defined by :

(a) $R(x, y, z) = \max[P(x, y), Q(y, z)]$

(b) $R(x, y, z) = \max \min[P(x, y), Q(y, z)]$

(c) $R(x, y, z) = \min[P(x, y), Q(y, z)]$

(d) None of the above

14. If ${}^{\alpha}A = [2, 4]$ and ${}^{\alpha}B = [3, 6]$, then the solution of equation ${}^{\alpha}A + {}^{\alpha}X = {}^{\alpha}B$ is :

(a) [1, 2]

(b) [4, –1]

(c) [–1, 4]

(d) None of the above

15. If $R_{12}(0, 0) = 0.9$, $R_{13}(0, 2) = 0.8$, $R_{23}(0, 2) = 0.2$, then cylindric closures Cyl (R_{12}, R_{13}, R_{23}) is :

(a) 0.9

(b) 1.9

(c) 0.2

(d) None of the above

16. If $P = \begin{bmatrix} .1 & .2 \\ .3 & 0 \end{bmatrix}$, $Q = \begin{bmatrix} .3 & .6 \\ 0 & .1 \end{bmatrix}$, then the value of P o Q is :

(a) $\begin{bmatrix} .4 & .8 \\ .3 & .1 \end{bmatrix}$

(b) $\begin{bmatrix} .1 & .2 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} .1 & .1 \\ .3 & .3 \end{bmatrix}$

(d) None of the above

17. If the basic assignment m specified by :

$$m = (0, .3, .4, 0, 0, .1, .2)$$

then possibility distribution r is :

(a) $r = (.2, .3, .3, .3, .7, 1, 1)$

(b) $r = (1, 1, .7, .3, .3, .3, .2)$

(c) $r = (.3, .1, .4, 0, .1, .1, .2)$

(d) None of the above

18. Which one of the following is correct :

(a) Bel $(A \cap B)$ = min [Bel (A), Bel (B)]

(b) Pl $(A \cup B)$ = max [Pl (A), Pl (B)]

(c) Bel (A) + Bel $(\overline{A}) \leq 1$

(d) All of the above

19. Every set $A \in P(X)$ is called a focal element of *m* if :

(a) $m(A) > 0$

(b) $m(A) \geq 0$

(c) $m(A) \leq 0$

(d) None of the above

20. The Dempster's rule of combination is expressed by the formula :

(a) $M_{1,2}(A) = \sum_{B \cup C = A} \frac{M_1(B), M_2(C)}{1 - K}$

(b) $M_{1,2}(A) = \sum_{B \cap C = A} \frac{M_1(B), M_2(C)}{K}$

(c) $M_{1,2}(A) = \sum_{B \cap C = A} \frac{M_1(B), M_2(C)}{1 - K}$

(d) None of the above

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Compute the scalar cardinality and relative cardinality of the fuzzy set :

$$A = \frac{.4}{v} + \frac{.2}{w} + \frac{.5}{x} + \frac{.4}{y} + \frac{1}{z}$$

2. Define convex fuzzy set.
3. Let $f : X \rightarrow Y$ be an arbitrary crisp function, for any $Ai \in \mathbf{J}(X)$ and $Bi \in \mathbf{J}(Y)$, $i \in I$. Then show that if $A_1 \subseteq A_2$, then $f(A_1) \subseteq f(A_2)$.
4. Define MIN and MAX for any two fuzzy numbers A and B.
5. Define *i* transitive closure.
6. Prove that :

$$(P \overset{i}{o} Q)^{-1} = Q^{-1} \overset{i}{o} P^{-1}$$

7. If $S(P, R) \neq Q$ then $\overset{\vee}{Q} = P^{-1} \overset{i}{o} R$ is the smallest member of S (P, R) where $P \overset{wi}{o} Q = R$.
8. Prove that :

$$P1(A) + P1(\overline{A}) \geq 1$$

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Prove that for every $A \in \mathbf{J}(X)$, $A = \bigcup_{\alpha \in [0,11]} {}_{\alpha}A$ where ${}_{\alpha}A = \alpha . {}^{\alpha}A$ and U denotes the standard fuzzy union.
2. Show that $u_w(a, c_w(a)) = 1$ for $a \in [0,1]$ all $w > 0$ where u_w and c_w denote the Yager union and complement respectively.
3. Prove that :

$$A(x) = \begin{cases} \sin x & 0 \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

is a fuzzy number.

4. Define max min composition on binary fuzzy relation.

5. The fuzzy relation R (X, X) represented by the membership matrix :

	a	*b*	*c*	*d*	*e*	*f*	*g*
a	0	.8	0	.4	0	0	0
b	.8	1	0	.4	0	0	0
c	0	0	1	0	1	.9	.5
d	.4	.4	0	1	0	0	0
e	0	0	1	0	1	.9	.5
f	0	0	.9	0	.9	1	.5
g	0	0	.5	0	.5	.5	1

Draw and explain partition tree for the similarity relation.

6. Prove that $i(a, b) \leq d$ iff $wi(a, b) \geq b$.

7. Show that every possibility measure PoS on a finite power set P (X) is uniquely determined by a possibility distribution function $r : \mathrm{X} \rightarrow [0,1]$ via the formula PoS (A) $\max_{x \in \mathrm{A}} r(x)$ for each $\mathrm{A} \in \mathrm{P}(\mathrm{X})$.

8. Let X = {*a*, *b*, *c*, *d*}. Given belief measure :

Bel ({*b*}) = .1

Bel ({*a*, *b*}) = 0.2

Bel ({*b*, *c*}) = .3

Bel ({*b*, *d*}) = 1.

Determine the corresponding basic assignment.

Section—D

5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Let A be a fuzzy set defined by :

$$A = \frac{.5}{x_1} + \frac{.4}{x_2} + \frac{.7}{x_3} + \frac{.8}{x_4} + \frac{1}{x_5}$$

Determine all α-cuts and strong α-cuts of A.

Or

Let C be a function from [0, 1] to [0, 1]. Then C is a fuzzy complement if and only if there exists a continuous function f from [0, 1] to R such that $f(1) = 0$, f is strictly decreasing and $c(a) = f^{-1}[f(0) - f(a)]$ for all $a \in [0, 1]$.

2. If :

$$A(x) = \begin{cases} 0 & \text{for } x < -2, x > 4 \\ \dfrac{x+2}{3} & -2 \le x \le 1 \\ \dfrac{4-x}{3} & 1 \le x \le 4 \end{cases}$$

$$B(x) = \begin{cases} 0 & x < 1, x > 3 \\ x - 1 & 1 \le x \le 2 \\ 3 - x & 2 \le x \le 3 \end{cases}$$

find MIN (A, B) (x) and MAX (A, B) (x).

Or

Let binary relation R (X, X) can be expressed by :

$$R(x, x) = \frac{.7}{(1,1)} + \frac{.3}{(1,3)} + \frac{.7}{(2,2)} + \frac{1}{(2,3)} + \frac{.9}{(3,1)} + \frac{1}{(3,4)} + \frac{.8}{(4,3)} + \frac{.5}{(4,4)}$$

Determine membership matrix, sagittal diagram and simple diagram.

3. Given :

$$Q = \begin{bmatrix} .1 & .4 & .5 & .1 \\ .9 & .7 & .2 & 0 \\ .8 & 1 & .5 & 0 \\ .1 & .3 & .6 & 0 \end{bmatrix}$$

and $r = .8 \quad .7 \quad .5 \quad 0$

Determine all solution of P o Q = R.

Or

Define fuzzy ordering relation.

4. Explain the concepts of fuzzy measures with examples.

Or

Explain fuzzy sets and possibility theory.