Roll No. .....

E–770

M. A./M. Sc. (Third Semester)
EXAMINATION, Dec.-Jan., 2020-21

MATHEMATICS

(Optional—B)

Paper Fifth

(Graph Theory—I)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. The maximum number of edges in a simple graph with n vertices is :

   (a) $\frac{n(n-1)}{2}$

   (b) $\frac{n-1}{2}$

   (c) $\frac{n}{2}$

   (d) $\frac{n+1}{2}$

2. A graph in which are vertices of equal degree is called :
   (a) Complete graph
   (b) Regular graph
   (c) Null graph
   (d) None of the above

3. Every null graph is regular of degree :
   (a) 2
   (b) 1
   (c) 0
   (d) None of the above

4. How many vertices and edges does the graph C_n have ?
   (a) n vertices and $n + 1$ edges
   (b) 2^n vertices and n edges
   (c) $(n + 1)$ vertices and n edges
   (d) n vertices and n edges

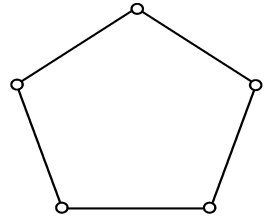
5. Isolated vertices are those with :
   (a) Zero degree
   (b) Degree one
   (c) Degree two
   (d) None of the above

6. Pendent vertex of degree :
   (a) Zero
   (b) One
   (c) Two
   (d) None of the above

7. A vertex with zero out degree is called :
   (a) Total degree
   (b) Source
   (c) Sink
   (d) None of the above

8. A graph with chromatic number less than or equal to *k* is called :
   (a) *k* chromatic colourable
   (b) *k* colourable chromatic
   (c) *k* chormatic
   (d) *k* colourable

9. What will be the chromatic number of the following graph ?



   (a) 3
   (b) 4
   (c) 5
   (d) 6

10. How many edges will a tree consisting of N modes have ?
   (a) N + 1
   (b) N – 1
   (c) N
   (d) log (N)

11. How many unique colours will be required for proper vector colouring of an empty graph having *n* vertices ?

    (a) *n*

    (b) 2

    (c) 1

    (d) 0

12. Minimum number of colours required for proper edges colouring of a graph is called :

    (a) Vertex matching

    (b) Chromatic index

    (c) Colour number

    (d) Chromatic number

13. Every graph of odd order is of class :

    (a) 0

    (b) 1

    (c) 2

    (d) 3

14. For any graph G :

    (a) $\alpha_0 + \beta_1 = \beta_0 + \alpha_1$

    (b) $\alpha_0 + \beta_0 = \alpha_1 + \beta_1$

    (c) $\alpha_0 + \alpha_1 = \beta_0 + \alpha_1$

    (d) $\alpha_0 \oplus \beta_0 = \alpha_0 \oplus \beta_1$

15. What is the definition of graph according to graph theory ?

    (a) Collection of vertices

    (b) Collection of edges

    (c) Collection of dots and lines

    (d) None of the above

16. Which one of the following is chromatic number of bipartite graph ?

    (a) 5

    (b) 4

    (c) 3

    (d) 1

17. Which of the following has maximum clique size ?

    (a) Perfect graph

    (b) Tree

    (c) Histogram

    (d) Cartesian

18. The number elements in the adjacency matrix of a graph having 7 vertices is :

    (a) 56

    (b) 49

    (c) 36

    (d) 14

19. In the given connected graph G, what is the value of rad (6) can diam (6) ?

    (a) 2, 2

    (b) 3, 3

    (c) 2, 3

    (d) 3, 2

20. Method of intersection is also known as :

(a) Recection

(b) Rediation

(c) Traversing

(d) Graphical triangulation

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Define topological operations.
2. Define vector space.
3. Explain spectrum properties.
4. Define cyclic graph.
5. Explain uniquely colourable.
6. Define achromatic and the adjoint number.
7. Explain clique parameters.
8. Explain circular arc graph.

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Define homomorphism and counteractions with an example.
2. Define adjacency matrix and vector space.
3. For any graph G, prove that :

$$\alpha_0 + \beta_0 = n$$

4. If G is a k-critical graph, then $\delta(G) \geq k - 1$.
5. Prove that every graph is an intersection graph.
6. Prove that the complement of every interval graph is a comparability graph.
7. Prove that any square submatrix of the adjacency matrix F of a graph G has determinant +1, –1 or zero.
8. Prove that the bond space β of G is the subspace R (δ) (the range of δ) of E. Then the stars span β and dimension $\beta = n - k = V$.

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Prove that if a graph is homeomorphic from graph G, then G is contraction of H.

Or

Prove that for any $S \geq 2$, $R(S, S) \geq 2^{S/2}$.

2. Prove that any graph G of order $n \geq 2$ without isolated vertices $\theta_1 \leq L\frac{n^2}{4}$ and the covering need use only edges and triangle.

Or

For every positive integer *k*, there is a *k*-chromatic triangle-tree graph.

3. Prove that the complement of every interval graph is a comparability graph.

Or

Prove that a graph G is a permutation graph iff G and $\overline{G}$ are comparability graphs.

4. Prove that every comparability graph is perfect.

Or

For any two positive integers $S_1, S_2 \geq 2$.