E–301

M. Sc. (First Semester)
EXAMINATION, Dec.-Jan., 2020-21

PHYSICS

Paper First

(Mathematical Physics)

Time : Three Hours]　　　　　　　[*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A　　　　1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. In vector spaces V (F) :

    (a)　An ordered double is a V (F)

    (b)　An ordered triple is a V (F)

    (c)　An ordered one is a V (F)

    (d)　None of the above

2. In transpose of matrix $(A^T)^T$ is equal to :

    (a)　A^T

    (b)　A

    (c)　A*

    (d)　None of the above

P. T. O.

3. Abelian group have properties :

    (a)   10

    (b)   5

    (c)   2

    (d)   3

4. The value of (A + B)* is equal to :

    (a)   A* + B*

    (b)   A*.B*

    (c)   A*B

    (d)   None of the above

5. The eigen vectors of a Hermitian matrix are :

    (a)   Real

    (b)   Imaginary

    (c)   Complex

    (d)   None of the above

6. Given two functions $u = x^2 - y^2$ and $v = \dfrac{y}{x^2 + y^2}$. Which of the following is true ?

    (a)   Both functions u and v are harmonic

    (b)   The function u is harmonic and v is not harmonic

    (c)   The function v is harmonic and u is not harmonic.

    (d)   Both functions u and v are not harmonic.

7. Which of the following equations is linear differential equation ?

    (a) $\dfrac{dy}{dx} + y \cos x = \sin x$

(b) $\dfrac{d^4 y}{dx^4} = [k + (y')^2]^{3/2}$

(c) $\dfrac{d^4 y}{dx^4} = \cos\left(\dfrac{dy}{dx}\right)$

(d)   None of the above

8. Logarithm of a complex number $z = (r, \theta)$ is a :
   (a)   Multivalued function
   (b)   Multivalued and single function
   (c)   Data insufficient
   (d)   Data sufficient

9. The general solution of a linear differential equation with constant coefficient is equal to :
   (a)   CF + PI
   (b)   CF – PI
   (c)   CF × PI
   (d)   None of the above

10. The analytic function $f(z)$ whose real part is $x^2 - y^2$ is :
   (a) z
   (b) z^2
   (c) z^{-2}
   (d)   None of the above

11. The value of magnitude of the integral $\displaystyle\int_C \dfrac{dz}{z}$ where C is $|z| = r$ is :
   (a) $2\pi r$
   (b) 2π
   (c) π
   (d)   None of the above

P. T. O.

12. In a Fourier series for a periodic continuous function b_n is :

   (a) $b_n = \dfrac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$

   (b) $b_n = \dfrac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$

   (c) $b_n = \pi \int_{-\pi}^{\pi} f(x) \sin nx \, dx$

   (d) $b_n = 2\pi \int_{-\pi}^{\pi} f(x) \sin x \, dx$

13. In Laplace transform value of e^{ax} is :

   (a) $\dfrac{1}{s - a}$

   (b) $\dfrac{1}{s}$

   (c) $\dfrac{1}{s + a}$

   (d) $\dfrac{1}{2s}$

14. In Laplace transform value of sin *at* is equal to :

   (a) $\dfrac{a}{s^2 + a^2}$

   (b) $\dfrac{a}{s^2}$

   (c) $\dfrac{s^2}{a^2}$

   (d) $\dfrac{a^2 + s^2}{a}$

15. According to Fourier expansion of x^2, the value of $\sum\limits_{n=2}^{\infty} \dfrac{1}{n^2}$ is

    :

    (a) $\dfrac{\pi^2}{3}$

    (b) $\dfrac{\pi^2}{4}$

    (c) $\dfrac{\pi^2}{6}$

    (d) $\dfrac{\pi^2}{12}$

16. Laplace transform of $\dfrac{1}{s\,(s^2 + 1)}$ is :

    (a) $(1 - \cos t)$

    (b) $(1 + \cos t)$

    (c) $(1 - \sin t)$

    (d) $(1 + \sin t)$

17. The eigen values of an antisymmetric matrix are :

    (a) ± 1

    (b) $\pm i$

    (c) Zero or imaginary

    (d) None of the above

18. Which of the following represent Fourier transform ?

    (a) $g\,(\alpha) = \int\limits_{0}^{\infty} f\,(t)\,e^{-\alpha t}\,dt$

(b) $g\ (\alpha) = \int_0^\infty f(t)\, e^{a-1} dt$

(c) Both (a) and (b)

(d) None of the above

19. The transform :

$$F(s) = \int_0^\infty F(t)\, e^{-ist} dt$$

is :

(a) Fourier transform

(b) Laplace transform

(c) Inverse Laplace transform

(d) None of the above

20. Classify the following differential equation :

$$w.\frac{dw}{dt} + 3t = 10$$

(a) Separable and not linear

(b) Both separable and not linear

(c) Linear and not separable

(d) Neither separable nor linear

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Define linearly dependent and linearly independent vectors.

2. Define vector subspaces.

3. What do you mean by Eigen value and Eigen vectors ?

4. What do you mean by order and degree of differential equation ? Define with *two* examples.

5. Explain the term singular point of an analytic function with example.

6. Explain Laguerre function.

7. What is Fourier series ?

8. Explain convolution theorem.

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Explain inner product of vectors space.

2. Explain unitary matrix with example.

3. Define a linear differential equation.

4. What do you mean by diagonalization of a matrix ?

5. Define residue of a pole.

6. Find the residue of :

$$f(z) = \frac{e^z}{z^2 + a^2}$$

7. State Cauchy-Riemann condition for analytic of complex variable in polar coordinate.

8. Prove that :

$$P_n\ x\ =\ -1\ ^n\ P_n\ -x$$

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Find the eigen values and eigen vectors of :

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

Or

Find the eigen values and eigen vectors of :

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

2. Apply the residue to show that :

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$$

Or

Apply the method of contour integration :

$$\int_0^{2\pi} \frac{1}{5 - 4\sin\theta}\, d\theta$$

3. Find the solution of the following differential equation :

$$\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2$$

when $y(0) = 0$ and $y'(0) = \dfrac{1}{2}$.

4. Find the Fourier integral of the functions :

$$f(x) = 0 \quad \text{when } x < 0$$
$$f(x) = 1 \quad \text{when } x = 0$$
$$f(x) = e^{-x} \quad \text{when } x > 0$$

Or

Find Laplace transform of :

$$F(t) = \int_0^t \frac{\sin u}{u}\, du$$