Roll No. ....

E–358

M. Sc. (IT) (First Semester) (ATKT) EXAMINATION, Dec.-Jan., 2020-21

Paper Second

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Time : Three Hours] [*Maximum Marks : 100*

[*Minimum Pass Marks : 40*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. Let A and B be sets and A^c and B^c denote the complements of the sets A and B. The set $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to :
   (a) $A \cap B$
   (b) $A^c \cap B^c$
   (c) $A^c \cup B^c$
   (d) $A \cup B$

2. The ‘Subset’ relation on a set of sets is :
   (a) A partial ordering
   (b) An equivalence relation
   (c) Transitive and symmetric only
   (d) Transitive and anti-symmetric only

3. $p \rightarrow q \;\wedge\; r \rightarrow q$ is equivalent to :
   (a) $p \vee \; r \rightarrow p$
   (b) $p \vee r \;\rightarrow q$
   (c) $p \vee \; r \rightarrow q$
   (d) $p \rightarrow \; q \rightarrow r$

4. Involution law is :
   (a) $p \vee q^{\;c} \equiv p^c \wedge q^c$
   (b) $\cdot \sim \; \sim p \; \equiv p$
   (c) $.p \vee p \equiv p$
   (d) $p \vee q \equiv q \vee p$

5. A self-complemented, distributive lattice is called :
   (a) Boolean Algebra
   (b) Modular lattice
   (c) complete lattice
   (d) Self dual lattice

6. Every finite subset of a lattice has :
   (a) may LUB’s and a GLB
   (b) A LUB and GLB
   (c) Many LUB’s and many GLB’s
   (d) Either some LUB’s or some GLB’s

7. NAND is a complement of :
   (a) AND
   (b) OR
   (c) XAND
   (d) NOT

8. The absorption law is defined as :
   (a) $.a * a = a$
   (b) $.a * 0 = 0$
   (c) $a * (a + b) = a$
   (d) $a + (a * b) = b$

9. A subgroup $(H, *)$ of (G, *) is called a normal subgroup of for any $a \in$ H, then ....
   (a) aH $\neq$ Ha
   (b) Ha = Ha^{-1}
   (c) Ha = aH
   (d) None of the above

10. The order of a subgroup of a finite group divides the order of the group. This theorem called :
   (a) Euler's theorem
   (b) Fermat's theorem
   (c) Cayley's representation theorem
   (d) Lagrange's theorem

11. Which of the following polynomials of degree 3 is a monic polynomials ?

(a) $3x^2 + 5x - 4 = 0$

(b) $2x^3 + 2x^2 + 2x + 4 = 0$

(c) $3x^3 + x^2 + 7 = 0$

(d) $4x^3 + 2x - 3 = 0$

12. Which of the following is true ?

(a) The set of all rational negative numbers forms a group under multiplication.

(b) The set of all non-singular matrices forms a group under multiplication.

(c) The set of all matrices forms a group under multiplication

(d) Both (b) and (c) are true

13. The minimum number of edges in a connected graph with n vertices.

(a) n

(b) $n + 1$

(c) $n - 1$

(d) None of the above

14. The total number of edges in a complex graph of n vertices is :

(a) $\frac{n}{2}$

(b) $n^2 - 1$

(c) $\frac{n^2 - 1}{2}$

(d) $\frac{n \quad n - 1}{2}$

15. In any undirected graph, the sum of degrees of all the vertices :

(a) must be odd

(b) must be even

(c) is twice the number of edges

(d) Both (b) and (c)

16. A directed graph G = (V, E) is said to be finite it its :

(a) Set V of vertices and set E of edges are finite

(b) Set V of vertices is finite

(c) Set E of edges is finite

(d) None of the above

17. A binary tree T has n leaf node. The number of nodes of degree 2 in T is ....

(a) $\log_2 n$ log

(b) 2^n

(c) $n-1$

(d) n

18. Which of the following traversal techniques lists the nodes of binary search tree in ascending order ?

(a) preorder

(b) Inorder

(c) postorder

(d) All of the above

19. The number of possible binary trees with 3 nodes is :
    (a) 5
    (b) 10
    (c) 12
    (d) 15

20. Preorder traversal is nothing but :
    (a) Depth of first order
    (b) Breadth first order
    (c) Topological order
    (d) Linear order

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions in 2-3 lines.

1. Let the functions f and g be defined by $f\ \ x\ \ = 2x + 1$ and $g\ \ x\ \ x^2 - 2$. Find the formula defining the composition function gof .
2. Find the truth tables for :
    (a) $p \vee \neg q$
    (b) $\neg p \wedge \neg q$
3. Define Sub lattices.
4. Define Boolean Algebra.
5. Define normal subgroup.
6. Define Minimal polynomials.
7. Define Psuedograph.
8. Define complete graph with example.
9. Draw all trees with exactly six vertices.
10. Define spanning tree.

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions within 75 words.

1. Verify that the proposition $p \wedge q \; \wedge \neg \; p \vee q$ is a contradiction.
2. Write difference between function and relation.
3. Define distributive lattice with example.
4. Draw the logic circuit with input a, b, c and output f where $f = abc + a'c' + b'c'$.
5. Define abelian group with example.
6. Define polynomial ring.
7. Define Hamiltonian circuit and Hamiltonian graph with example.
8. Explain the path and connectivity in directed graph.
9. Write the applications of trees in computer sciences.
10. Explain complete and extended binary trees.

Section—D 6 each

(Long Answer Type Questions)

Note : Attempt all questions within 150 words.

1. Show that the relation "$xRy \Leftrightarrow x - y$ is divisible by 3" when $x, y \in \mathrm{I}$ defined in the set of integers I is an equivalence relation.

Or

Is the following argument valid ?

If two sides of a triangle are equal,

then the opposite angles are equal,

Two sides of a triangle are not equal,

$\therefore$ The opposite angles are not equal.

2. Prove that for any *a* and *b* in a Boolean algebra (B, $\vee$, $\wedge'$,),

   (a) $(a \vee b)' = a' \wedge b'$ and

   (b) $(a \wedge b)' = a' \vee b'$

Or

Show that dual of a lattice is a lattice.

3. Prove that a subset $s \neq \phi$ of G is a subgroup of (G, *) iff for any pair of elements $a, b \in S; a * b^{-1} \in S$.

Or

Prove that in a ring (R, +, .)

(i) $a.0 = 0.a = 0 \; \forall \; a \in R$

(ii) $a.(-b) = (-a).b = -(a.b) \;\; \forall a, b \in R$

4. Explain Dijkstra algorithm with example.

Or

Write applications of Graph theory in computer science.

5. Explain traversing binary trees.

Or

Prove that every connected graph has at least one spanning tree.