Roll No. ....

E–313

M. A./M. Sc. (First Semester) EXAMINATION, Dec.-Jan., 2020-21

MATHEMATICS

Paper Fifth

(Advanced Discrete Mathematics—I)

Time : Three Hours] [*Maximum Marks : 80*

[*Minimum Pass Marks : 16*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. The proposition $p \wedge (\sim p \vee q)$ is :

   (a) A tautology

   (b) A contradiction

   (c) Logically equivalent to $p \wedge q$

   (d) None of these

2. Let p denote "He is rich" and let q denote "He is happy". Write "He is neither rich nor happy" statement in symbolic form using p and q. Which of the following symbolic form is correct ?

   (a) $p \to \neg q$

   (b) $\neg p \to \neg q$

   (c) $q \to \neg p$

   (d) $\neg p \leftrightarrow \neg q$

3. A semigroup < M > with an identity element with respect to operation (.) is called :

   (a) Monoid

   (b) Homomorphism

   (c) Automorphism

   (d) None of these

4. Every finite semigroup has an :

   (a) Identity element

   (b) Inverse element

   (c) Idempotent element

   (d) None of these

5. Let M be the set of all $n \times n$ matrices and let the binary operation * of M be taken as addition of matrices. Then (M, *) is a :

   (a) Semigroup

   (b) Monoid

   (c) Both (a) and (b)

   (d) None of these

6. An equivalent relation R on a semigroup (S, *) is called a congruence relation if :

   (a) $a\mathrm{R}b'$ and $b\mathrm{R}a' \Rightarrow (a * b')\ \mathrm{R}\ (a' * b)$

   (b) $a'\mathrm{R}b$ and $b'\mathrm{R}a \Rightarrow (a' * b)\ \mathrm{R}\ (a * b')$

   (c) $a\mathrm{R}b$ and $a'\mathrm{R}b' \Rightarrow (a * b)\ \mathrm{R}\ (a' * b')$

   (d) $a\mathrm{R}a'$ and $a\mathrm{R}b' \Rightarrow (a * b)\ \mathrm{R}\ (a' * b')$

7. Union of two sub-semigroups of a semigroup (S, *) is :

   (a) Semigroup of (S, *)

   (b) Sub-semigroup of (S, *)

   (c) Sub-monoid of (S, *)

   (d) Need not be a sub-semigroup of (S, *)

8. Which of the following statements is a proposition ?

   (a) Get me a glass of milk.

   (b) What is your name ?

   (c) The only odd prime number is 2.

   (d) God bless you !

9. Every finite subset of a lattice has :

   (a) A LUB and GLB

   (b) Many LUB's and a GLB

   (c) Many LUB's and many LGB'S

   (d) Either some LUB's or some GLB's

10. A self-complemented, distributive lattice is called :

   (a) Modular lattice

   (b) Boolean algebra

   (c) Complete lattice

   (d) Self-dual lattice

11. The term sum-of-product in Boolean algebra means :

(a) AND function of several OR functions

(b) OR function of several AND functions

(c) AND function of several AND functions

(d) None of these

12. The Boolean expression A + BC equals :

(a) $(A + B)(A + C)$

(b) $(A + B)(\overline{A} + C)$

(c) $(\overline{A} + B)(\overline{A} + C)$

(d) $(A + \overline{B})(A + \overline{C})$

13. The dual of Boolean expression $(a + 1)(a + 0) = a$ is :

(a) $a + 0 = a$

(b) $a.0 + a.1 = a$

(c) $a + 1 = a$

(d) $a.(1 + 0) = a$

14. If $< T, *, \oplus >$ is lattice and if $S \subseteq T$, then $< S, *, \oplus >$ is sublattice of $< T, *, \oplus >$ if and only if :

(a) S is closed under the operation $\oplus$

(b) S is closed under the operation (*)

(c) S is associative under the operation (*)

(d) S is closed under operations (*) and $\oplus$

15. How many truth tables can be made from one function table ?

(a) 1

(b) 2

(c) 3

(d) 4

16. In a lattice property $a \vee a = 0, a \wedge a = a$ is called :

(a) Idempotent Laws

(b) Commutative Laws

(c) Absorption Laws

(d) None of these

17. Let L be a language recognizable by a finite automation. The language REVERSE (L) = {*w* such that *w* is the reverse of *v* where $V \in L$ } is a :

(a) Regular language

(b) Context-free language

(c) Context-sensitive language

(d) None of these

18. A regular grammar contains only productions of the form $\alpha \to \beta$, where :

(a) $|\alpha| \leq |\beta|$

(b) $|\alpha| < |\beta|$

(c) $|\alpha| > |\beta|$

(d) $|\alpha| \geq |\beta|$

19. Which of the following regular expression identifiers are true ?

(a) $(r + s)^* = r^* + s^*$

(b) $r^*.s^* = r^* + s^*$

(c) $(r^*)^* = r^*$

(d) All of these

20. If L (a) = $\{a^p : p \text{ is prime}\}$, then :

    (a) L (a) is regular

    (b) L (a) is reduced grammar

    (c) L (a) is not regular

    (d) None of these

Section—B $1\frac{1}{2}$ each

(Very Short Answer Type Questions)

Note : Attempt all questions. Answer in 2-3 sentences.

1. Define Tautology.
2. Define semi-group with *one* example.
3. Define submonoids.
4. Give example of a sub-semigroup.
5. Define distributive lattices.
6. Define sublattice.
7. Define minterm or minimal boolean function.
8. Define join-irreducible elements.
9. Define context-free grammar.
10. Define type zero grammar.

Section—C $2\frac{1}{2}$ each

(Short Answer Type Questions)

Note : Attempt all questions. Answer in less than 75 words.

1. Define contradiction. Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is contradiction.
2. Consider the set Q of rational numbers, and let * be the operation on Q defined by :

$$a * b = a + b - ab$$

   Is (Q, *) a semigroup ? Is it commutative ?

3. Define direct product of semigroups.
4. Define monoid homomorphism.
5. Draw the logic circuit with inputs a, b, c and output f where :

$$f = abc + a'c' + b'c'$$

6. Prove that let $(L, \leq)$ be a lattice for any $a, b, c \in L$, the following holds :

$$a \leq c \Rightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c$$

7. Simplify the Boolean expression :

$$E(x_1 x_2) = x_1 x_2 + x_1' x_2' + x_1' x_2$$

8. Show that the order relation $\leq$ is partial order relation in a Boolean algebra.
9. Define the following terms :
   (a) Context-sensitive grammar
   (b) Regular grammar
10. Show that the language :

$$L(G) = \{a^n b^n c^n : n \geq 1\}$$

can be generated by G = (N, T, P, S) where N (S, B, C); T = (a, b, c). P = ($S \to asBc, S \to aBc, cb \to Bc, aB \to ab, bB \to bb, bc \to bc, cc \to cc$) and S is the starting symbol.

Section—D 4 each

(Long Answer Type Questions)

Note : Attempt all questions. Answer using less than 150 words for each.

1. What is Quantifiers ? Explain different types of quantifiers.

Or

Show that the following argument is not valid :

$$\begin{array}{l} p \\ \neg q \vee r \\ \underline{\neg p \Rightarrow q} \\ r \end{array}$$

where $\neg$ = negation.

2. State and prove fundamental theorem of homomorphism for semigroup.

Or

Prove that if (S, *) and (T, *′) are monoids, then (S × T, *′) is also a monoid where binary operation *″ defined on S × T by :

$$(s_1, t_1) *'' (s_2, t_2) = (s_1 * s_2, t_1 *' t_2)$$

$$\forall \ (s, t) \text{ and } (s_2, t_2) \in S \times T$$

3. In a Boolean algebra (B, +, .); state and prove :

(a) Absorption law

(b) De Morgan's law

Or

Prove that the direct product of any two distributive lattices is a distributive lattice.

4. Show that the algebra of Boolean circuits is a Boolean algebra.

Or

Draw the switching circuit of the function :

$$f(x, y, z) = x.y'(z + x) + y.(y' + z)$$

and replace it by a simplified one.

5. State and prove pumping lemma for regular sets.

Or

Construct a grammar for the language :

$$L = \{a^m b^m : n \neq m, n > 0\}$$