Roll No. ....

E–3919

B. C. A. (Part III) EXAMINATION, 2021

(Old Course)

Paper First

CALCULUS AND GEOMETRY

(301)

Time : Three Hours] [*Maximum Marks : 50*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Let $f \in R[a, b]$ and let *m*, M be bounds of *f* on [*a*, *b*]. Then :

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a) \text{ if } b \geq a$$

$$m(b-a) \geq \int_a^b f(x)\,dx \geq M(b-a) \text{ if } a \geq b.$$

(b) State and prove fundamental theorem of integral calculus.

P. T. O.

(c) Let $f(x) = \sin x$ for $x \in \left[0, \frac{\pi}{2}\right]$ and let

$P = \left\{0, \frac{\pi}{2n}, \frac{2\pi}{2n}, \ldots\ldots, \frac{n\pi}{2n}\right\}$ be the partition of $\left[0, \frac{\pi}{2}\right]$.

Compute U (P, *f*) and L (P, *f*). Hence prove that :

$$f \in R\left[0, \frac{\pi}{2}\right].$$

Unit—II

2. (a) Discuss maxima, minima and saddle point for the function :

$$f(x, y) = x^3 - 4xy + 2y^2.$$

(b) Discuss maxima or minima of :

$$u = \sin x \sin y \sin(x + y).$$

(c) Determine maximum and minimum value of the function :

$$u = x^2 + y^2 + z^2,$$

under the conditions :

$$ax^2 + by^2 + cz^2 = 1$$

and

$$lx + my + nz = 0.$$

Unit—III

3. (a) Test the convergence of integral :

$$\int_{a}^{\infty} \frac{dx}{x^n}$$

where $a > 0$.

(b) Test the convergence of the integral :

$$\int_{a}^{b} \frac{dx}{(x-a)\sqrt{b-x}}.$$

(c) Test the convergence of the integral :

$$\int_{0}^{\infty} \frac{\cos x}{1+x^2}\, dx.$$

Unit—IV

4. (a) Find the equation of a cone whose vertex is at (α, β, γ) and base curve is :

$$ax^2 + by^2 = 1, \; z = 0.$$

(b) Find the equation of a right circular cone whose vertex is at origin, axis is $x = y = z$ and half vertex angle is 45°.

(c) Find the equation of cylinder whose generator line is parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and passes through the curve $x^2 + y^2 = 16, \; z = 0$.

Unit—V

5. (a) Show that the condition that the straight line $\frac{l}{r} = A\cos\theta + B\sin\theta$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$ is $(A - e)^2 + B^2 = 1$.

(b) Convert $(x-2)^2 + (y-3)^2 = 13$ to polar form.

(c) Convert the equation of straight line $y = mx + c$ into polar form.