**Roll No. ..................................**

# E–3826

## M. A./M. Sc. (Final) EXAMINATION, 2021

MATHEMATICS

**(Compulsory)**

Paper First

**(Integration Theory and Functional Analysis)**

*Time : Three Hours* ] [ *Maximum Marks : 100*

**Note :** Attempt any *two* parts from each question. All questions carry equal marks.

**Unit—I**

1. (a) State and prove Hahn decomposition theorem.

 (b) State and prove Lebesgue decomposition theorem.

 (c) Define product measure. Let $\{(A_i \times B_i)\}$ be a countable disjoint collection of measurable rectangle whose union is a measurable rectangle $A \times B$. Then :

$$\lambda (A \times B) = \Sigma \lambda (A_i \times B_i)$$

## Unit—II

2. (a) Define regular measure. Prove that the intersection of a sequence of inner regular set of finite measure is inner regular. Also, the intersection of a decreasing sequence of outer regular set of a finite measure is outer regular.

(b) State and prove Riesz-Markoff theorem.

(c) If a function $f$ is absolutely continuous in an open interval $(a, b)$ and if $f'(a) = 0$ a.e. in $[a, b]$, then $f$ is constant.

## Unit—III

3. (a) Prove that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.

(b) Let X be a normed linear space. Then show that the closed unit ball $B = \{x \in X : \|x\| \le 1\}$ in X is compact if and only if X is finite dimensional.

(c) Let $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then prove that $l_p{}^*$ is isometrically isomerophic to $l_q$.

## Unit—IV

4. (a) State and prove uniform boundedness theorem.

(b) Let X and Y be Banach space and T be a continuous linear transformation of X onto Y, then the image of each open sphere centred on the origin in X contain an open sphere centred on the origin in Y.

(c) Define reflexive space. Prove that if X is a Banach space, then X is reflexive if and only if X* is reflexive.

**Unit—V**

5. (a) Let C be a non-empty closed and convex set in an Hilbert space H, then there exists a unique vector in C of smallest norm.

(b) Prove that every Hilbert space H is reflexive.

(c) Let T be a bounded self-adjoint operator on a real Hilbert space H, then :

$$\|T\| = \text{Sup} \ |< x, Tx| : \|x\| = 1$$