Roll No. ....

E–3822

M. A./M. Sc. (Previous) EXAMINATION, 2021

MATHEMATICS

Paper Second

(Real Analysis)

Time : Three Hours] [*Maximum Marks : 100*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) If f be a bounded function and α be a monotonically increasing function on [a, b], then $f \in R(\alpha)$ if and only if for every $\in z_0$ there exists a partition set P such that :

$$U(P, f, \alpha) - L(P, f, \alpha) < \in$$

P. T. O.

(b) If α be a monotonically increasing function on [*a*, *b*] and $\alpha' \in R[ab]$. Let *f* be a bounded real function on [*ab*], then $f \in R(\alpha)$ if and only if $f\alpha' \in R[a,b]$ and $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.

(c) If $f(x) = x$ and $\alpha(x) = x^2$, does $\int_0^1 f d\alpha$ exist ? If it exists, then find its value.

Unit—II

2. (a) Prove that the series $1 - 1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + ...$ is convergent and its sum is zero while the sum of the rearranged series $1 + \frac{1}{2} - 1 + \frac{1}{3} + \frac{1}{4} - \frac{1}{2} + ...$ is log 2, and of series $1 + \frac{1}{2} + \frac{1}{3} - 1 + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{2} + ...$ is log 3.

(b) Explain pointwise convergence and uniform convergence of sequences with examples.

(c) A series of function $\sum_{n=1}^{\infty} u_n(x)$ will converge uniformly on X of there exists a convergent series $\sum_{n=1}^{\infty} M_n$ of positive constants such that :

$$|u_n(x)| \le M_n \quad \text{for all } n \text{ and } x \in X.$$

Unit—III

3. (a) Define linear transformation and if a linear operator A on a finite-dimensional rector space X is one-to-one if and only if the range of A is all of X, that is, iff A is onto.

   (b) State and prove inverse function theorem.

   (c) State and prove Stokes' theorem.

Unit—IV

4. (a) Define outer measure and prove that the outer measure of an interval is its length.

   (b) State and prove Lebesgue differentiation theorem.

   (c) Define Lebesgue measurable set and prove that if E_1 and E_2 are measurable sets, then so $E_1 \cup E_2$.

Unit—V

5. (a) Let $1 \leq p < \infty$ and let $f, g \in L^p(u)$. Then $f + g \in L^p(u)$ and :

$$\| f + g \|_p \leq \| f \|_p + \| g \|_p$$

   (b) State and prove Jensen's inequality.

   (c) Define L^p space and prove that L^p space is complete.