Roll No. ....

E–3823

M. Sc./M. A. (Previous) EXAMINATION, 2021

MATHEMATICS

Paper Third

(Topology)

Time : Three Hours]　　　　　　　　[*Maximum Marks : 100*

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define cardinality. Prove that $n + \alpha = \alpha$, $\forall n \in N$, α being any infinite cardinal number.

   (b) Define closure of a set. Let A, B be subsets of a topological space. Then prove that :

   (i) A is closed in X if and only if $\overline{A} = A$

   (ii) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

   (c) Define sub-base for a topological space with an example.

   Let (X, **T**) be a topological space and **S** be a family of subsets of X. Then prove that **S** is a sub-base for **T** if and only if **S** generates **T**.

Unit—II

2. (a) Prove that every second countable space is first countable but the converse need not be true.

   (b) Prove that every Tychonoff space is a T_3-space.

   (c) Define Hausdorff space. Prove that the property of being a T_2-space is a topological invariant property.

Unit—III

3. (a) Prove that a topological space is compact if and only if every family of closed subsets of it which has the finite intersection property has a non-empty intersection.

   (b) Let **C** be a collection of connected subsets of a topological space (X, **T**) such that no two member of are mutually separated. Then prove that $\bigcup_{C \in \mathbf{C}} C$ is also connected.

   (c) Prove that a metric space is compact if and only if it is complete and totally bounded.

Unit—IV

4. (a) Prove that the product space $X_1 \times X_2$ is compact if and only if each of the spaces X_1 and X_2 is compact.

   (b) State and prove Tychonoff Embedding theorem.

   (c) Let $X = \prod_{i \in I} X_i$ and $x \in X$. Then prove that X is first countable at x if and only if for each $i \in I$, X_i is first countable at $\pi_i(x)$ and for all except countably many i's X_i is the only neighbourhood of $\pi_i(x)$ in X_i.

Unit—V

5. (a) Prove that every filter is contained in an ultra filter.

   (b) If X is a path connected space, then prove that for any pair of points x_0 and x_1 in X, $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic.

   (c) Define Net and its convergence. Prove that a topological space (X, **T**) is Hausdorff if and only if every net in X can converge to atmost one point.