Roll No. ....

E–3834

M. Sc./M. A. (Final)

EXAMINATION, 2021

MATHEMATICS

(Optional)

Paper Fifth (*iii*)

(Fuzzy Sets and Their Applications)

Time : Three Hours] [*Maximum Marks : 100*

Note : Attempt any *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) Define and find some examples of :

(i) Interval valued fuzzy sets

(ii) Ordinary fuzzy set

(iii) L-fuzzy sets

(iv) Type two fuzzy sets

(b) State and prove first decomposition theorem.

(c) For all $a, b \in \; 0,1$, prove that :

$$i_{\min} \;\; a,b \;\; \leq i \;\; a,b \;\; \leq \min \;\; a,b$$

where $i_{\min}$ denotes the drastic intersection.

Unit—II

2. (a) If $f : \mathrm{X} \rightarrow \mathrm{Y}$ is a crisp function and $\mathrm{A}_i \in \mathrm{F} \;\; x$, $i \in \mathrm{I}$, then prove that :

(i) $$\mathrm{F}\left(\bigcup_{i \in \mathrm{I}} \mathrm{A}_i\right) = \bigcup_{i \in \mathrm{I}} f \;\; \mathrm{A}_i$$

(ii) $$f\left(\bigcap_{i \in \mathrm{I}} \mathrm{A}_i\right) \subseteq \bigcap_{i \in \mathrm{I}} f \;\; \mathrm{A}_i$$

(b) Prove that max-min composition and min-join composition are associative operations on binary fuzzy relations.

(c) Explainfuzzy equivalence relation with an example.

Unit—III

3. (a) Solve the following fuzzy relation equation for the max-min composition :

$$p_o \begin{bmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{bmatrix} = \;\; .6 \, .6 \, .5$$

(b) Prove that every possibility measure Pos on a finite power set P(X) is uniquely determined by a possibility distribution function :

$$r : x \to \ 0,1$$

via the formula Pos (A) = $\max_{x \in A} r \ \ x$

for each A $\in$ P x .

(c) Let X = a, b, c, d . Given the belief measure Bel ({b}) = 0.1, Bel ({a, b}) = 0.2, Bel ({b, c}) = 0.3, Bel ({b, d}) = 1, determine the corresponding basic assignment.

Unit—IV

4. (a) Write a short note on linguistic variables and hedges.

(b) Explain unconditional and unqualified fuzzy propositions with suitable examples.

(c) Suppose we have a fuzzy conditional and qualified proposition.

p : If *x* is A, then *y* is B is very true, where :

$$A = \frac{1}{x_1} + \frac{.5}{x_2} + \frac{.7}{x_3}$$

$$B = \frac{.6}{y_1} + \frac{1}{y_2}$$

and S stands for every true; let S (a) = a^2 for all $a \in [0, 1]$. Given a fact "x is A", where $A' = \frac{.9}{x_1} + \frac{.6}{x_2} + \frac{.7}{x_3}$, then using method of truth-value restrictions conclude that "y is B″.

Unit—V

5. (a) Explain general scheme of a fuzzy controller.

(b) Assume that each individual of a group of five judges has a total preference ordering $p_i \; i \in N_5$ on four figure skaters a, b, c, d. The orderings are : $p_1 = \langle a, b, d, c \rangle$, $p_2 = \langle a, c, d, b \rangle$, $p_3 = \langle b, a, c, d \rangle$, $p_4 = \langle a, d, b, c \rangle$, $p_5 = \; c, a, b, d$. Use fuzzy multiperson decision-making to determine the group decision.

(c) Solve the following fuzzy linear programming problem :

$$\max = 6x_1 + 5x_2$$

such that :

$$\langle 5, 3, 2 \rangle x_1 + \langle 6, 4, 2 \rangle x_2 \le \langle 25, 6, 9 \rangle$$

$$\langle 5, 2, 3 \rangle x_1 + \langle 2, 1.5, 1 \rangle x_2 \le \langle 13, 7, 4 \rangle$$

$$x_1, x_2 > 0 .$$