Roll No. .....

E–996

M. A./M. Sc. (Fourth Semester) (Main/ATKT) EXAMINATION, May-June, 2021

MATHEMATICS

Paper Third (C)

(Mathematical Biology—II)

Time : Three Hours] [*Maximum Marks : 80*

[*Minimum Pass Marks : 16*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose one correct answer out of four alternative answers :

1. Assume that a cohort of members of the population born at the time c. Then the age of any member of this cohort at time t is :

   (a) $t + c$

   (b) $t - c$

   (c) $t \times c$

   (d) None of the above

2. Which of the following is known as Von Foerster equation ?

   (a) $\rho_a(a,t) + \rho_t(a,t) + \mu(a)\rho(a,t) = 0$

   (b) $\mu(a)\rho_a(a,t) + \rho_t(a,t) + \rho(a,t) = 0$

   (c) $\rho_a(a,t) + \mu(a)\rho_t(a,t) + \rho(a,t) = 0$

   (d) None of the above

3. Kernel of the given equation

$$B(t) = \psi(t) + \int_0 t\beta(a)\pi(a)B(t-a)\,da$$

   is :

   (a) $\psi(t)\pi(a)$

   (b) $\psi(t)$

   (c) $\beta(a)\pi(a)$

   (d) None of the above

4. Which of the following is true for an equilibrium age distribution ?

   (a) It is solution of $\rho(a,t)$ that is independent of *t*.

   (b) At an equilibrium age disribution McKendrick equation becomes an ordinary differential equation.

   (c) Both (a) and (b) are true.

   (d) None of the above

5. Which of the following is true for the model equation ?

$$p' = cp(1 - p) - ep$$

(a) If $c < e$, then only equilibrium is $p = 0$.

(b) If $c > e$, then there is an asymptotically stable equilibrium $p \neq 0$.

(c) Both (a) and (b) are true.

(d) None of the above

6. Equation $u_t(x, t) = \mathrm{D}u_{xx}(x, t)$ is known as :

(a) Heat equation

(b) Diffusion equation

(c) Both (a) and (b)

(d) None of the above

7. Which of the following is true about solving diffusion equation by separation of variables ?

(a) We look for the solution of the form $u(x, t) = \mathrm{X}(x)\mathrm{T}(t)$.

(b) Diffusion equation is transformed to a pair of partial differential equations.

(c) Both (a) and (b)

(d) None of the above

8. Which of the following equations belongs to diffusion in one dimension ?

(a) $u_t(x, t) = \mathrm{D}u_{xx}(x, t)$

(b) $u_t(x, t) = \mathrm{D}u_{xx}(x, t) + g(u)$

(c) Both (a) and (b)

(d) None of the above

9. Diseases that are always present in a community, usually at a low, more or less constant, frequency are classified as having an ..... pattern.

   (a) epidemic

   (b) endemic

   (c) pandemic

   (d) None of the above

10. An epidemic that becomes unusually widespread and even global in its reach is referred to as a .....

   (a) hyperendemic

   (b) pandemic

   (c) Spanish flu

   (d) None of the above

11. Which of the following statements is true concerning epidemic diseases ?

   (a) They are usually not very contagious.

   (b) They usually appear and disappear seasonally.

   (c) At the end of an epidemic, a disease spreads at an increasing rate and then abruptly disappears.

   (d) None of the above

12. An epidemic will die out when the reproductive number is :

   (a) 1

   (b) > 1

   (c) > 2

   (d) < 1

13. Duration of infectiousness can be reduced by :

    (a) Wearing mask

    (b) Imposing lockdown

    (c) Using suitable medicine

    (d) None of the above

14. In the SIR epidemic model, mean infection period is :

    (a) α

    (b) $\frac{1}{\alpha}$

    (c) $\beta - \alpha$

    (d) None of the above

15. If in a closed population an epidemic spreads, the number of susceptibles will be :

    (a) Increase

    (b) Decrease

    (c) Stabilize

    (d) None of the above

16. If a vaccinated person got infected, then its infectivity will be :

    (a) Reduced

    (b) Increased

    (c) Unchanged

    (d) None of the above

17. Which of the following models has control reproduction number ?
    (a) SIR model
    (b) Quarantine-isolation model
    (c) Both (a) and (b)
    (d) None of the above

18. The incubation period is the interval between :
    (a) The time of infection and death
    (b) The time of infection and appearance of clinical symptoms
    (c) Time of infection and appearance of antibodies
    (d) None of the above

19. Which of the following is a vector-borne disease ?
    (a) influenza
    (b) meningitis
    (c) tuberculosis
    (d) None of the above

20. What does the reproduction number of a virus tell us ?
    (a) How many contacts will be infected from one case
    (b) Length of the 'incubation period'
    (c) Length of infection
    (d) None of the above

Section—B 2 each

(Very Short Answer Type Questions)

Note : Answer all questions in 2-3 sentences.

1. Define birth modulus.
2. What do you mean by metapopulation ?

3. Write down non-linear reaction diffusion equation.
4. Write down relation for herd immunity.
5. Define control reproduction number.
6. Draw flow chart for the Influenza model.
7. What do you mean by spectral radius of a matrix ?
8. What do you mean by temporary immunity ?

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Describe a metapopulation model with two patches and no internal patch dynamics.
2. Give sematic diagram for disease model with exposed period and obtain expression for reproduction number.
3. Discuss SEIR model as an age of infection model.
4. Write down assumptions and model equations for treatment model.
5. Draw flow chart for a vaccination model. Write down assumptions and model equations.
6. Define herd immunity. Obtain expression for herd immunity for vaccine with efficacy $\in$.
7. Using first generation matrix obtain basic reproduction rate for the following model :

$$S' = \beta S\left(I + \in E\right)$$

$$E' = \beta S\left(I + \in E\right) - \kappa E$$

$$I' = \kappa E - \alpha I$$

$$R' = \alpha I$$

8. Write a note on diseases as population control.

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Explain method of characteristics for structured population model.

   Or

   Discuss in detail mathematical model for age structured linear continuous model.

2. Derive expression for diffusion in two dimensions.

   Or

   Find a solution of the diffusion equation :

   $$u_t(x,t) = \mathrm{D}u_{xx}(x,t) \text{ for } 0 \le x \le \mathrm{L}, 0 \le t < \infty,$$

   subject to the boundary conditions $u(0,t) = u_x(\mathrm{L},t) = 0$ and the initial condition $u(x,0) = u_0\delta(x - x_0)$.

3. Discuss in detail the SIR epidemic model.

   Or

   Explain Quarantine-Isolation model.

4. Discuss SIR model with temporary immunity.

   Or

   Explain the SIR model with births and deaths.