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M.Sc. (IT) (First Semester)
EXAMINATION, Dec. - Jan., 2021-22

Paper Third

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Time : Three Hours] *[Maximum Marks:100*

[Minimum Pass marks :40

Note : Attempt all section as directed

Section - A
(Objective/Multiple Choice Questions)

(1 mark each)

Note: Attempt all questions.

1. Consider the following relation on {1,2,3,4,5,6}.

   $R = \{(i.j): |i - j| = 2\}$, then *R* is:

   (A) Reflexive

   (B) Symmetric

   (C) Transitive

   (D) All the above

2. Idempotent law is:

   (A) $p \wedge p \equiv p$

   (B) $p \vee p \equiv p$

   (C) Both (A) and (B)

   (D) None of the above

3. Let P = (1,2,3) and Q = (1,a,b). Then cardinal number of the set $P \times Q, i.e. |P \times Q|$ is

   (A) 3

   (B) 6

   (C) 9

   (D) None of the above

4. $p \vee q$ is logical equivalent to

   (A) $\sim q \to \sim p$

   (B) $q \to p$

   (C) $\sim p \to \sim q$

   (D) $\sim p \to q$

5. Which of the following one does not hold in Boolean algebra.

   (A) Involution law

   (B) Absorption law

   (C) Cancellation law

   (D) Uniqueness of complement.

6. The possible number of combinations of minterms of n variables are:

   (A) 2^n

   (B) n^2

   (C) 2^{n-1}

   (D) None of the above

7. A self complemented, distributive lattice is called:

   (A) Boolean algebra

   (B) Complete lattice

   (C) Modular lattice

   (D) None of the above

8. Every finite lattice is:
   (A) Bounded
   (B) Unbounded
   (C) Undefined
   (D) None of the above
9. Consider the set of natural numbers $\mathbb{N}$, then which one is true.
   (A) $(\mathbb{N},-) \, is \, group$
   (B) $(N,+) \, is \, group$
   (C) $(N,\times) \, is \, group$
   (D) None of the above
10. Identity permutation is an:
   (A) Even permutation
   (B) Odd permutation
   (C) Both (A) and (B)
   (D) None of the above
11. Let $G=\{1,-1,i,-i\}$ be a multiplicative group, then the order of the element $i, i.e. \, o(i)$ is
   (A) 1
   (B) 2
   (C) 3
   (D) 4
12. Let $(G,+_6)$ is a cyclic group. where G = {0,1,2,3,4,5}. Then
   (A) 1 is a generator of cyclic group
   (B) 5 is a generator of cyclic group
   (C) Both (A) and (B)
   (D) None of the above

13. A complete graph K_n is planar if
   (A) $n=5$
   (B) $n<5$
   (C) $n>5$
   (D) None of the above
14. Let *G* be a connected planar graph with 8 vertices, 12 edges, and 2 regions, then the sum of all degrees of regions is:
   (A) 22
   (B) 24
   (C) 16
   (D) None of the above
15. A complete bipartite graph $K_{m,n}$ is planar if
   (A) $m<3 \, or \, n<3$
   (B) $m>3 \, or \, n>3$
   (C) $m>3 \, or \, n<3$
   (D) None of the above
16. Hamilton cycle of graph *G* is a cycle that contains every____of graph *G*.
   (A) Path
   (B) Cycle
   (C) Vertex
   (D) Edge

17. A tree has two vertices of degree 2, One vertex of degree 3 and three vertices of degree 4. How many vertices of degree 1?
    (A) 9
    (B) 5
    (C) 10
    (D) 7
18. The number of pendant vertices in a full binary tree with n vertices is
    (A) $\frac{n-1}{2}$
    (B) $\frac{n+1}{2}$
    (C) $\frac{n-2}{2}$
    (D) $\frac{n}{2}$
19. The number of different spanning tree of the complete graph K_n is
    (A) 2^n
    (B) n^2
    (C) n^{n-2}
    (D) None of the above
20. Under which condition the complete bipartite graph $K_{m,n}$ becomes tree
    (A) $m = 1 \, or \, n = 1$
    (B) $m = 2 \, or \, n = 2$
    (C) $m = 3 \, or \, n = 3$
    (D) None of the above

Section - B

(Very Short Answer Type Question)

(2 marks each)

Note: Attempt all questions.

1. Define universal quantifier with example.
2. Define equivalence relations.
3. Define distributive lattice.
4. Define Boolean algebra with example.
5. Define normal subgroup.
6. Define cosets.
7. Define planar graph with example.
8. Define Euler graph with example.
9. Define spanning tree.
10. Write the statemnt of Euler's formula.

Section - C

(Short Answer Type Questions)

(3 marks each)

Note: Attempt all questions.

1. Construct truth table for the following functions and check whether it is a tautology or contradiction.

   $$\left[(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)\right] \Leftrightarrow \left[(p \vee q) \wedge (q \vee r) \wedge (r \vee p)\right]$$
2. Show that the argument $p.p \rightarrow q \vdash q$ is vaild.
3. Show that a lattice L is modular if for any $a, b, c \in L$ the following relation holds. $a \vee (b \wedge (a \vee c)) = (a \vee b) \wedge (a \vee c)$
4. Express the following Boolean function as a product of maxterms. $f(a,b,c) = ab + a'c.$

5. Prove that the intersection of any two subgroup of a group is also a subgroup of *G*.

6. Examine whether the following permuation is even or odd.

$$\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 2\,5\,4\,3\,6\,1\,7\,9\,8 \end{pmatrix}$$

7. A planar simple graph G has 30 vertices, each of degree 3. Determine the number of regions into which this planar graph can be splitted.

8. Prove that in a complete graph G with n (odd integer greater than or equal to 3) vertices there are $\frac{n-1}{2}$ edge disjoint Hamiltonian circuits.

9. Show that in any tree there are at least two pendant vertices.

10. Prove that a graph with vertices, $n-1$ edges and having no circuits is a connected.

Section - D

(Long Answer Type Questions)

(6 marks each)

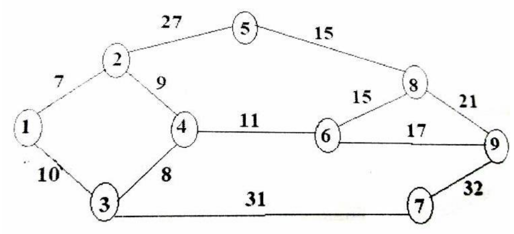
Note: Attempt all questions.

1. Show that $p \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r.$

2. Show that every chain is a distributive lattice.

3. Let H be a subgroup of a finite group G. Then prove that the number of left and right cosets of H in G are equal.

4. Find the shortest path from Node 1 to Node 9 in the follwing weighted graph:



5. Determine the minimal spanning tree for the graph given below.