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M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

PAPER FIRST

(Advanced Abstract Algebra)

Time : Three Hours] *[Maximum Marks:100*

Note : Attempt any two parts from each question. All questions carry equal marks.

Unit - 1

1. (A) State and prove Jordan-Holder Theorem for finite group.

(B) Prove that every subgroup of solvable group is solvable.

(C) Define algebraic extension of a field and show that every finite extension of a field is an algebraic extension.

Unit - 2

2. (A) Find the Galois group of $x^3 - 2 \in Q[x]$.

(B) State and prove Artin theorem.

(C) Show that the polynomial $x^5 - 9x + 3$ is not solvable by radical over Q.

Unit - 3

3. (A) Prove that the necessary and sufficient condition for an R-module M to be a direct sum of its two submodules N_1 and N_2 are that

(i) $M = N_1 + N_2$ and,

(ii) $N_1 \cap N_2 = \{0\}$

(B) Prove that every submodules and every quotient modules of a noetherian module is noetherian.

(C) State and prove Hilbert basis theorem.

Unit - 4

4. (A) Let U and V be two vector spaces over a field F of dimensions m and n respectively. Then prove that Hom (U V) is a vector space over F of dimension mn.

(B) Let $\lambda_1, \lambda_2, \lambda_3 ....\lambda_k \in F$ be distinct characteristics roots of $T \in A$ (V) and let v_1, v_2, ...v_k be characteristic vector of T belonging to $\lambda_1, \lambda_2, \lambda_3 ....\lambda_k$ respectively. Then prove that v_1, v_2, ...v_k are linearly independent over F.

(C) Prove that the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

is nilpotent and find its invariants and Jordan form.

Unit - 5

5. (A) Obtain the smith normal form and rank for the following matrix over Q [*x*]

$$\begin{bmatrix} -x-3 & 2 & 0 \\ 1 & -x & 1 \\ 1 & -3 & -x-2 \end{bmatrix}$$

(B) State and prove fundamental structure theorem for finitely generated modules over a principal ideal domain.

(C) Find rational canonical form of the following matrix over Q

$$A = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & -3 & -2 \end{bmatrix}$$