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F-3955

B.C.A. (PART II) Examination, 2022

(Old Course)
Paper Second
DIFFERENTIATION AND INTEGRATION
(201)

Time : Three Hours]   *[Maximum Marks : 50*

Note : Attempt any two parts from each question. All questions carry equal marks.

Unit - I

1. (a) State and prove Leibnitz's theorem.

    (b) If $a, b \in R$ such that $a \neq b$, then show that there exists a real number $c \in (a, b)$ such that
    $$a^2 + ab + b^2 = 3c^2$$

    (c) Expand log sin x in powers of (x – 2).

Unit - II

2. (a) Prove that radius of curvature at the point (x, y) of the catenary $y = c \cosh\left(\dfrac{x}{c}\right)$ is $\dfrac{y^2}{c}$.

    (b) Find Points of inflexion of the curve y = 3x³ – 4x² + 1.

    (c) Trace the curve r = a (1 + cos θ)

Unit - III

3. (a) If $u = x^2 + y^2 + z^2$, then prove that
    $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2u$$

    (b) Find the directional derivative of $\Phi = x^2 yz + 4xz^2$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$ at the point (1, –2, –1).

    (c) If u = $x + y - z$, v = $x - y + z$ and $w = x^2 + y^2 + z^2 - 2xyz$, then show that
    $$\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$$

Unit - IV

4. (a) Evaluate $\int \dfrac{x\,dx}{(x-1)(x-2)(x-3)}$

(b) Find the value of $\int_1^3 \dfrac{\cos(\log x)}{x}\,dx$

(c) Prove that $\lim\limits_{n \to \infty} \left(\dfrac{n^n}{n!}\right)^{1/n} = e$

Unit – V

5. (a) Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)\,dz\,dy\,dx$.

(b) Change the order of integration in the following

integral $\int\limits_0^a \int\limits_{\sqrt{ax-x^2}}^{\sqrt{ax}} V(x,y)\,dx\,dy$

(c) Find the area of the common region of the two curves $y^2 = ax$ and $x^2 + y^2 = 4ax$.