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# F - 3856

## M.A./M.Sc. (Final) Examination, 2022
## MATHEMATICS
## (Compulsory)
### Paper First
### (Integration Theory and Functional Analysis)

*Time : Three Hours]* *[Maximum Marks:100*

**Note: Attempt any two parts from each questions. All questions carry equal marks.**

### Unit - 1

1. (a) Let E be a measurable set of finite measure, that is $0 < \mu(E) < \infty$. Then prove that E contains a positive set A with $\mu(A) > 0$.





(b) State and prove Radon Nikodym theorem.

(c) State and prove Riesz representation theorem.

### Unit - 2

2. (a) Show that every compact Bair set is $G_\delta$ type.

(b) If $\mu$ is finite Baire measure on the real line. Then prove that its cumulative distribution function F is monotone increasing bounded function which is continuous on right and $\lim_{x \to -\infty} F(x) = 0$

(c) Let $\mu$ be a measure defined on $\sigma$ - algebra A containing the Baire sets. Assume that either $\mu$ is quasi regular or $\mu$ is inner regular. Then prove that for each E $\in$ A with $\mu(E) < \infty$, there is a Baire set B with $\mu(E \text{V} B) = 0$

### Unit - 3

3. (a) State and prove Riesz lemma.

(b) Prove that in a finite dimensional normed linear space, all the norms are equivalent.

(c) Let M be a closed linear subspace of a norm linear space X. Then prove that the quotient space X/M is a norm linear space with the norm $\|x+M\| = \inf\{\|x+m\| : m \in M\}$. Further prove that if x is Banach space, then so X/M.

**Unit - 4**

4. (a) State and prove closed graph theorem.

(b) State and prove Hahn Banach theorem for real linear space.

(c) Let {Tn} be a sequence of compact linear operators from a normed space X into a Banach space Y and T be a bounded linear operator $T : X \to Y$, such that $\|Tn - T\| \to 0$ as $n \to \infty$, then prove that limit operator T is compact.

**Unit - 5**

5. (a) State and prove projection theorem.

(b) Prove that a normed space is an inner product space if and only if the norm of the normed space satisfies the parallelogram law.





(c) Prove that the self adjoint operator in B(H) form a closed linear subspace and therefore a real Banach space which contains the identity transformation.