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M. A./M.Sc. (Final) Examination, 2022

Mathematics

(Optional)

Paper Third (i)

Graph Theory

Time : Three Hours] *[Maximum Marks:100*

Note: Attempt **any two** parts from each question. All questions carry equal marks.

1. (A) Prove that any homomorphism is the product of a connected and a discrete homomorphism.

   (B) Prove that if G is a k-regular graph, k is an Eigen value of G. This is simple if G is connected. Every other Eigen value has absolute value $\leq k$

   (C) Prove that any square sub matrix of the adjacency

matrix F of a graph G has determinant +1, -1 or zero.

2. (A) Prove that any uniquely k - colourable graph is (k -1) connected.

   (B) Prove that for any graph G with $\delta > 0$,

   $\alpha_1 + \beta_1 = n$

   (C) For any graph G of order $n \geq 2$ without isolated vertices, $\barwedge_1 \leq \lfloor n^2/4 \rfloor$ and the partition need use only edges and triangles. Prove.

3. (A) Prove that a graph is triangulated iff every minimal vertex-separator induces a complete sub graph.

   (B) Prove that the complement of every interval graph is a comparability graph.

   (C) Prove that a graph G is a permutation graph iff G and $\overline{G}$ are comparability graphs.

4. (A) Prove that every group is isomorphic to the automorphism group of some graph.

   (B) Prove that if the Eigen values of the digraph D are all distinct, then T (D) is abelian.

   (C) Prove that each cycle $C_n,\ n \geq 3$ is chromatically unique.

5. (A) Prove that every digraph without odd cycles has a 1- basis.

   (B) Prove that a weak digraph is strong iff each of its blocks is strong.

   (C) State and prove vertex form of Menger's theorem.