Roll No. .... Total Printed Pages - 11

F - 521

M.A./M.Sc.(Second Semester)
EXAMINATION, May-June, 2022
MATHEMATICS
Paper Third
(General and Algebraic Topology)

Time : Three Hours] *[Maximum Marks: 80*

Section - A

(Objective/Multiple Choice Questions)

(1 mark each)

Note : Attempt all questions.

Choose the correct answer:

1. A completely regular T-,space is called.......
   (A) Normal space
   (B) Regular space
   (C) Tychonoff space
   (D) Hausdorff space

2. The product of finitely many compact space is
   (A) Compact space
   (B) Open set
   (C) Null set
   (D) None of these
3. A countable product of first countable space is
   (A) First countable
   (B) Second countable
   (C) Third countable
   (D) Fourth countable
4. A subset of R^n is closed and bounded iff it is compact. This theorem is known as:
   (A) Tychonoff theorem
   (B) Urysohn metrization theorem
   (C) Projection theorem
   (D) Generalised Heine-Borel theorem
5. A topological space is said to be T_u-space if it is........
   (A) Regular and T_1
   (B) Completely regular and T_1
   (C) Normal and T_1
   (D) None of these

6. A completely normal space which is also T_1 is called.....
   (A) T_2-space
   (B) T_3-space
   (C) T_4-space
   (D) T_5-space
7. The product space X_1 x X_2 is connected iff
   (A) X_1 is connected
   (B) X_2 is connected
   (C) Both X_1 and X_2 are connected
   (D) None of these
8. Which one is not a correct statement-
   (A) A product is first countable iff each product co-ordinate space is first countable and all except finitely many co-ordinate spaces are indiscrete.
   (B) A topological product is second countable iff all co-ordinate spaces are so and except countable many are indiscrete spaces
   (C) Let Y be seprable and let I=[01] then product Y^I is not separable
   (D) Product of spaces is totally disconnected iff each co-ordinate space is so

9. Which of the following is false?
   (A) Every closed subspace of a para compact space is para compact
   (B) Every para compact space is normal
   (C) An arbitrary space of a para compact space and product of para compact space need not be para compact
   (D) Every metrizable space need not be para compact
10. Let x be a metrizable space. Then x has basis that is.....
   (A) Uncountable locally finite
   (B) Countable locally finite
   (C) Countable locally Infinite
   (D) Uncountable locally infinite
11. Which of the following is not an example of locally finite?
   (A) $u=\{(n, n+z):n \in z\}$
   (B) $u_1=\{(0,1/n):n \in z\}$
   (C) $B=\{(n,2n):n \in z\}$
   (D) $B_1=\{(n,5n:n \in z\}$
12. Every regular Lindeloff space is ....
   (A) Para compact
   (B) Sequently compact
   (C) Locally compact
   (D) Countable compact

13. Let x={abc}.Then which of following is not a filter

(A) F_1={x}

(B) F_2={{a,b},x}

(C) F_3={{a},{a,b}{a,c}x}

(D) F_4={{a},{b},{a,b},x}

14. Which of the following is not true?

(A) Two filter bases B_1 and B_2 on x are said to be equivalent iff they generate the same filter on x

(B) If B is a filter base on X. A filter F on X is called filter generated by B if the member of F contains a member of B

(C) A filter base on set X is called ultrafilter base iff it is base of an ultrafilter

(D) If F is a filter on X and $A \subset X$ then F is said to be eventually in a iff $A \not\subseteq F$.

15. Which of the following is not true?

(A) $(N, \geq)$ is a directed set

(B) $(R, \geq)$ is not a directed set

(C) Every residual subset of A is a cofinal subset of A

(D) Every cofinal subset of A is directed by the relation $\geq$

16. A net in set X is a function f: $A \rightarrow X$ where A is.........

(A) Directed set

(B) Residual subset

(C) Cofinal subset

(D) None of these

17. The fundamental group π (S'$_0$ b_0) of circle S' is isomorphic to:

(A) Multiplicative group of integers

(B) Additive group of integers

(C) Additive group modulo(m) of integers

(D) None of these

18. Every polynomial of n degree has exactly

(A) (n-1) roots

(B) (n-2) roots

(C) n roots

(D) 1 root

19. Let $x_0, x_1 \in X$. If there is a path in X from x_0 to x_1 then the group $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are:

(A) Isomorphic

(B) Homomorphic

(C) Endomorphic

(D) Homotopy

20. Which of the following is not true

(A) A covering mapping is a local homeomorphism

(B) A covering mapping is open

(C) A covering mapping is onto

(D) A local homeomorphism is covering map

Section - B

(Very Short Answer Type Questions)

(1.5 marks each)

Note: Attempt all questions using 2-3 sentences.

1. Define wall
2. Define evaluation mapping
3. Define finitely short of topological space.
4. State Alexander sub-base theorem.
5. Define metrizable topological space.
6. Define locally finite of topological space
7. Define cluster point of a net
8. Define ultra filter
9. Define Homotopy of paths
10. Define covering mapping.

Section - C

(Short Answer Type Questions)

(2.5 marks each)

Note : Attempt all questions precisely using less than 75 words.

1. Explain box and wall in cartesian product of spaces.
2. Prove that projection map is continuous
3. Prove that product space of two Hausdorff space is Hausdorff.
4. Let {(X,d) be a metric space and let λ be any positive real number. Then there exist a metric e on X such that $e(x,y) \leq \lambda$} for all $x,y \in X$ and e induces the same topology or X as d does.
5. Let $\{f_i : X \rightarrow Y_i \mid i \in I\}$ be a family of functions which distinguishes points from closed sets in X. Then the corresponding evaluation function e: $X \rightarrow \pi\, Y_i$ is open when regarded as $i \in I$ function from X onto ecx)
6. Every tychonoff space X can be embedded as a subspace of a cube
7. A topological space is Hausdorff iff every net in X can converge to at most one point.

8. Let $\{F_\lambda : \lambda \in \wedge\}$ be any non empty family of filters on a non empty set X. Then the set $F = \cap\{F_\lambda : \lambda \in \wedge\}$ is also a filter on X.

9. If h: $(X, x_0) \rightarrow Q(Y, y_0)$ Then $(koh)^* = k^* oh^*$. If $I:(X, x_0) \rightarrow (X, x_0)$ is the identity map, then i^* is the identity homeomorphism.

10. If X is locally connected then a Continuous map $P:\overline{X} \rightarrow X$ is a covering map iff for each component H of X. the map $P/p^{-1}(H): p^{-1}(H) \rightarrow H$ is covering map

Section - D

(Long Answer Type Questions)

(4 marks each)

Note:- Attempt all questions precisely using 150 words.

1. Let (X,T) be the product space of (X_1, T_1) and (X_2, T_2). Let $\pi_1 : X \rightarrow X_1, \pi_2 : X \rightarrow X_2$ be the projection maps on first and second co-ordinate spaces respectively. Let $f : Y \rightarrow X$ be another map where Y is another topological space. Show that *f* is continuous iff π_1 of and π_2 of are continuous maps.

OR

The product space $X = \pi\{X_i : i \in I\}$ is a T_1-space iff each co-ordinate space is T_1

P.T.O.

2. A product space is locally connected iff each co-ordinate space is locally connected and all except finitely many of them are connected.

OR

State and prove Tychonoff's Theorem.

3. Let $\{f_i : X \rightarrow Y_i \, \forall i \in I\}$ be a family of continuous function which distinguishes points and also distinguishes points from closed sets, then the corresponding evaluation mapping is an embedding of X into the product space $\underset{i \in I}{\pi} Y;$

OR

Let X be a regular space with a basis B that is countably locally finite. Then X is metrizable.

4. Define convergence of net and let (X T) be a topological space and $Y \subset X$ then show that Y is T-open iff no net in X-Y can converges to a point in Y

OR

For a filter F on a set X the following statements are equivalent:

(i) F is an ultrafilter

(ii) For any $A \subset X$ either $A \in F$ or $X-A \in F$

(iii) For any $AB \subset X$, $A \cup B \in F$ iff either $A \in F$ or $B \in F$

5. If f, g and h are three paths such that f*g and g*h exist, then (f*g)*h and f*(g*h) exist and $(f*g)*h \sim f*(g*h)$

OR

A polynomial equation $x^n+a_{n-1}x^{n-1}+...+a_1x+a_0=0$ of degree n>0 with real or complex co efficients has at least one root.