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M.A./M.Sc. (Second Semester) EXAMINATION, MAY-JUNE, 2022

MATHEMATICS

Paper Fourth

[Advanced Complex Analysis (II)]

Time : Three Hours] *[Maximum Marks : 80*

Note : Attempt all sections as directed.

(Section-A)

(Objective/Multiple Choice Questions)

(1 mark each)

Note- Attempt all questions.

Choose the correct answer :

1. Value of G(z) is

(A) $\prod_{n=1}^{\infty}\left(1+\frac{z}{n}\right)e^{-z/n}$

(B) $\prod_{n=1}^{\infty}\left(1+\frac{z}{n}\right)e^{z/n}$

(C) $\prod_{n=1}^{\infty}\left(1-\frac{z}{n}\right)e^{-z/n}$

(D) None of these

2. The entire function e^z have

(A) One zero

(B) n-zeros

(C) No zeros

(D) Infinite zeros

3. Which one is the pole of Gamma function ΓZ ?

(A) 1

(B) 2

(C) 3

(D) 0

4. Euler's Gamma function is meromorphic with poles at

(A) Non-negative integers

(B) Non-positive integers

(C) Both (A) and (B)

(D) None of these

5. If $f_2(z)$ is an analytic continuation of $f_1(z)$ from domain D_1 into D_2, then $D_1 \cap D_2 =$

(A) ϕ

(B) Non empty

(C) Complex plane

(D) None of these

6. The purpose of analytic continuation is to-

(A) Enlarge the domain

(B) Shrink the domain

(C) Restrict the domain

(D) None of the above

7. These cannot be more than one analytic continuation of a function f(z) in the same domain.

(A) True

(B) False

8. The Poisson Kernel $P_r(\theta) = \sum_{n=-\infty}^{\infty} r^{|n|} e^{in\theta}$, $-\infty < \theta < \infty$ is defined, when

(A) $r < 0$

(B) $0 \le r < 1$

(C) $-\infty < r < \infty$

(D) None of these

9. Which of the following statement is false?

(A) ϕ is superharmonic iff -ϕ is subhormonic

(B) Every harmonic function is subharmonic

(C) Every harmonic function is superharmonic

(D) None of these

10. Germ of a function is defined as

(A) A function itself

(B) A domain of a function

(C) Collection of function elements

(D) None of these

11. A harmonic function is

(A) A closed map

(B) an open map

(C) Both (A) and (B)

(D) None of these

12. If G is a region such that no component of $\mathbb{C}_\infty - G$ reduces to a point, then G is

(A) Dirichlet region

(B) Connected region

(C) Barrier

(D) None of these

13. An entire function f(z) is said to be of infinite order, if for sufficiently large value of r

(A) $M(r) \le exp(r^\lambda)$

(B) $M(r) \ge exp(r^\lambda)$

(C) $M(r) > exp(r^\lambda)$

(C) $M(r) < exp(r^\lambda)$

14. If f(z) is an entire function of order λ and convergence exponent σ, then

(A) $\sigma \le \lambda$

(B) $\lambda \le \sigma$

(C) $\sigma = \lambda$

(D) None of these

15. If p is the rank of *f* and *q* is the degree of the polynomial *g*, then genus of *f* is

(A) $\mu = \min(p, q)$

(B) $\mu = max(p, q)$

(C) $\mu < max(p, q)$

(D) $\mu < min(p, q)$

16. Order of polynomial $p(z) = a_0 + a_1 z + \cdots\cdots\cdots + a_n z^n$, $a_n \ne 0$ is

(A) 0

(B) 1

(C) 2

(D) ∞

17. The derivative of a univalent function is

(A) Zero

(B) Non-zero

(C) Not a constant

(D) None of these

18. Γz is not defined at

(A) $z=1$

(B) $z=0$

(C) $z=1/2$

(D) $z=-1/2$

19. A univalent function that maps $|z|<\infty$ auto $|\omega|<\infty$ must be

(A) Constant

(B) Zero

(C) Linear

(D) Non-linear

20. A univalent map of the extended plane onto the extended plane must be

(A) Linear

(B) Bilinear

(C) Non-linear

(D) None of these

(Section- B)

(Very Short Answer Type Questions)

(2 marks each)

Note- Attempt all questions.

1. Define entire function.

2. Define Gamma function.
3. State Monodromy theorem.
4. Define Green's function.
5. Define fixed-end point homotopy.
6. State little picard theorem.
7. Define univalent functions.
8. Define Poisson-Kernel.

(Section - C)

(Short Answer Type Questions)

(3 marks each)

Note- Attempt all questions.

1. Prove that $\sqrt{\pi}\Gamma(2z)=2^{2Z-1}\Gamma(z)\Gamma(z+\frac{1}{2})$
2. State and prove Euler's theorem.
3. Show that the series $\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}}$ and $\sum_{n=0}^{\infty}\frac{(z-i)^n}{(2-i)^{n+1}}$ are analytic continuation of each other.
4. Let $U:G\to R$ be a continuous function which has the mean value property, then *u* is harmonic.
5. Let G be a region and let a $a\in\partial_\infty G$ such that there is a barrier for G at a, if $f:\partial_\infty G\to\mathbb{R}$ is continuous and u is

the perron function associated with f, then $\lim_{z \to a} u(z) = f(a)$

6. Let f(z) be analytic in the closed dise $|z| \le R$.

Assume that $f(0) \neq 0$ and no zero of f(z) lie on $|z| = R$. If $z_1, z_2, ..., z_n$ are the zeros of f(z) in the open disc $|z| < R$, each repeated as often as its multiplicity, then

$$\log|f(0)| = -\sum_{i=1}^{n} \log\left(\frac{R}{|z_i|}\right) + \frac{1}{2\pi}\int_0^{2x} \log\left(f(\mathrm{Re}^{i\phi}\right) d\phi$$

7. State and prove Hadamard's factorization theorem.

8. Let f be analytic in $D = \{z : |z| < 1\}$ and let $f(0) = 0$, $f'(0) = 1$ and $|f(z)| \le m$ for all z in D. then $M \ge 1$ and $f(D) \supset B\left(0; \frac{1}{6M}\right)$

Section D

(Long Answer Type Questions)

(5 marks each)

Note- Attempt all questions.

1. State and prove Weierstrass factorization theorem.

OR

For $\mathrm{Re}\, Z > 1$,

$$\xi(z)\Gamma z = \int_0^{\infty} (e^t - 1)^{-1} t^{z-1} dt.$$

2. State and prove Schwarz's Reflection Principle.

OR

State and Prove Hornack's theorem.

3. State and prove Hadamard's there circles theorem.

OR

State and prove Schottky's theorem.

4. State and prove Montel Coratheodory theorem.

OR

State and prove Great Picard theorem.