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M.A./M.Sc. (Fourth Semester)
EXAMINATION, MAY-JUNE, 2022
MATHEMATICS
PAPER SECOND
PARTIAL DIFFERENTIAL EQUATIONS AND MECHANICS-II

Time : Three Hours] *[Maximum Marks : 80*

Note : Attempt all sections as directed.

(Section-A)

(Objective/Multiple Choice Questions)

(1 mark each)

Note- Attempt all questions.

Choose correct answer.

1. The non-linear wave equations is the PDE-

(A) $u_t - \Delta u = f(u)$

(B) $u_{tt} - u_x = f(u)$

(C) $u_{tt} - div\vec{a}(Du) = 0$

(D) $u_t - div(Du) = 0$

2. $x\,Du + f(Du) = u$ is known as-

(A) Heat equation

(B) Wave equation

(C) Porous medium equation

(D) Clairaut's equation

3. Second order parabolic PDE is of form-

(A) $u_t + \Delta u = 0$

(B) $u_{tt} + u_x = 0$

(C) $u_t + u_{xx} = 0$

(D) $u_{tt} + Du = 0 \; in \; U_T$

4. The IVP for Burger's equation is-

(A) $u_{tt} + u_x = 0 \; and \; u = g \; on \; RX\{t = 0\}$

(B) $u_t + \Delta u = 0 \; and \; u = g \; on \; RX\{t = 0\}$

(C) $u_t + \left(\frac{u^2}{2}\right)_x = 0 \, , u = g \; on \; RX\{t = 0\}$

(D) $u_{tt} + \left(\frac{u^2}{2}\right)_x = 0 \, , u = g \; on \; RX\{t = 0\}$

5. Equation $u(x,t) = v(x - \sigma t)(x \in R, t \in R)$ is known as-

(A) Travelling wave

(B) Exponential equation

(C) KdV equation

(D) Telegraph equation

6. Which result is true-

(A) $\int_{R^n} e^{-bx^2} dx = \left(\frac{\pi}{b}\right)^n$

(B) $\int_{R^n} e^{-bx^2} dx = \left(\frac{\pi}{b}\right)^{n/2}$

(C) $\int_{R^n} e^{-bx} dx = \left(\frac{\pi}{b}\right)^n$

(D) $\int_{R^n} e^{-bx^2} = \left(\frac{\pi}{b}\right)^{n/2}$

7. The transformation $\omega = e^{-b/a u}$ is known as-

(A) Fourier transform

(B) Legendre transform

(C) Laplace transform

(D) Cole-Hopf transformation

8. 1-D Telegraph equation is given by-

(A) $u_{xx} + 2d\ u_x - u_t = 0$

(B) $u_{tt} + 2d\ u_t - u_{xx} = 0$

(C) $u_{tt} + 2d\ u_x - u_{xx} = 0$

(D) None of these

9. Expansion is known as $f = \sum_{\alpha} f_{\alpha} . x^{\alpha}$

(A) Power series

(B) Multi-indices

(C) Majorizes

(D) None of the above

10. The PDE is known as $u_{tt} - \sum_{k,l=1}^{n} a^{kl}(x).u_{x_k x_l} = 0\ in\ R^n X(0,\infty)$

(A) Hyperbolic equation

(B) Parabolic equation

(C) Elliptic equation

(D) Spherical equation

11. Taylor expansion about x_0 when $|x - x_0| < r$ is

(A) $f(x) = \sum_{\alpha} f(x - x_0)^{\alpha}$

(B) $f(x) = \sum_{\alpha} \frac{1}{L\alpha} f(x - x_0)^{\alpha}$

(C) $f(x) = \sum_{\alpha} \frac{1}{L\alpha} D^{\alpha} f(x_0).(x - x_0)^{\alpha}$

(D) $f(x) = \sum_{\alpha} D^{\alpha} f(x).(x - x_0)^{\alpha}$

12. The j[th] normal derivative of u at $x^0 \in$ is

(A) $\frac{\partial^j u}{\partial v^j} = \sum_{|\alpha|=j} \binom{j}{\alpha} D^{\alpha} u.v^{\alpha}$

(B) $\frac{\partial^j u}{\partial v^j} = \sum_{|\alpha|=j} D^{\alpha} u.v^{\alpha}$

(C) $\frac{\partial^j u}{\partial v^j} = \sum_{|\alpha|=j} D^{\alpha} u.D^{\alpha} v$

(D) $\frac{\partial^j u}{\partial v^j} = \sum_{|\alpha|=j} \binom{j}{\alpha} D^{\alpha} u.v$

13. The following differential equations are known as-

$$\frac{dq_j}{dq_1} = \frac{\partial K}{\partial P_j}, \frac{\partial P_j}{\partial q_1} = -\frac{\partial K}{\partial q_j} \quad (j = 2,3,\ldots.,n)$$

(A) Euler equation

(B) Jacobi equation

(C) Whittaker's equation

(D) Hamilton's principle

14. The transformation $\alpha = aq + bp$, $P = cq + dp$ is canonical if

(A) ad + bc =1

(B) ad = bc = 0

(C) ad - bc = 0

(D) ad - bc = 1

15. For generating function $F_2 = \sum q_i P_i$, which result is true-

(A) $P_i = P_i$, $q_i = Q_i$

(B) $P_i = -P_i$, $q_i = Q_i$

(C) $P_i = P_i$, $q_i = -Q_i$

(D) $P_i = -P_i$, $q_i = -Q_i$

16. The generating function for the transformation $P = \frac{1}{Q}, P = \frac{q}{Q^2}$ is given by

(A) $F = \frac{q^2}{Q}$

(B) $F = \frac{p^2}{Q}$

(C) $F = \frac{P}{Q}$

(D) $F = \frac{q}{Q}$

17. If $S(q_i, \alpha_i, t)$ for i=1,2,....,n be any integral of the equation

$\frac{\partial S}{\partial t} + H\left(\frac{\partial S}{\partial q_i}, q_i, t\right) = 0$ is-

(A) First form of Jacobi's equation

(B) Second form of Lagrange's equation

(C) First form of Lagrange's equation

(D) Second form of Lagrange's equation

18. The correct relation between the variation is-

(A) $\Delta q_r = \dot{q}_r \Delta t + \delta q_r$

(B) $\delta q_r = \dot{q}_r \Delta t + \Delta q_r$

(C) $\dot{q}_r = \Delta q_r \Delta t + \delta q_r$

(D) $q_r = \Delta q_r + \delta q_r . \Delta t$

19. Hamilton's characterstic function W(q,p) satisfying the equation-

(A) $H\left(\frac{\partial \omega}{\partial q_i}, q_i\right) = -\alpha_1$

(B) $H\left(\frac{\partial \omega}{\partial q_i}, q_i\right) = \alpha_1$

(C) $H\left(\frac{\partial \omega}{\partial q_i}, q_i\right) = 0$

(D) $H\left(\frac{\partial \omega}{\partial q_i}, q_i\right) \neq \alpha_1$

20. For Hamiltonian $H = \frac{1}{2}(q^2 + p^2)$

(A) $[\dot{p}, H] = p$

(B) $[\dot{p}, H] = q$

(C) $[\dot{q}, H] = q$

(D) $[\dot{q}, H] = -q$

(Section- B)

(Very Short Answer Type Questions)

(2 marks each)

Note : Attempt all questions.

1. Define complete integral of non-linear first order PDE: F(Du, u, x)=0
2. Write Hamilton-Jacobi equation
3. Define Fourier transform.
4. Write fundamental solution of heat equation.
5. Define majorizes of power series.

6. Define Real analytic functions.
7. Define Poisson brackets.
8. Write statement for second form of Jacobi's theorem.

(Section - C)

(Short Answer Type Questions)

(3 marks each)

Note : Attempt all questions.

1. Explain the Rarefaction wave.
2. Define Riemann Problem.
3. Write any three properties of Fourier transform.
4. Explain Cauchy data and non characteristic surface for the PDE.
5. Derive Hamilton's Principle from Newton's Equations.
6. Verify whether or not the transformation $P = \frac{1}{2}(p^2 + q^2),\ Q = \tan^{-1}\frac{q}{p}$ is a contact transformation?
7. Prove that the Lagrange's bracket does not obey the

commutative law of algebra.

8. Write a short note on separation of variables in Hamilton-Jacobi equation.

Section D

(Long Answer Type Questions)

(5 marks each)

Note:- Attempt any four questions.

1. State and prove local existence theorem for nonlinear first order partial differential equation.

2. State and prove Lax-Oleinik formula.

3. State and prove Plancherel's theorem.

4. Write a short note on Hodograph and Legendre transform.

5. State and prove Cauchy-Kovalevskaya theorem.

6. Derive Whittaker's equations.

7. The transformation equations between two sets of coordiantes are

$Q = \log(1+\sqrt{q}.\cos p),\ P = 2(1+\sqrt{q}\cos p)\sqrt{q}\sin p$

Show that these transformations are canonical if q and p are canonical.

8. Discuss motion of a particle falling under gravity, using Hamilton-Jacobi equation.