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# F - 997

## M.A/M.Sc. (Fourth Semester)
## EXAMINATION, May - June, 2022
## MATHEMATICS
## (Optional-A)
## Paper Fourth
## (Operations Research-II)

*Time : Three Hours]* *[Maximum Marks:80*

**Note: Attempt all sections as directed.**

**(Section-A)**

**(Objective Type Questions)**

**(1 mark each)**

**Note: Attempt all questions :**

1. When a positive quantity *n* is divided into *c* parts, the maximum value of their product is :

   (A) $\left(\frac{n}{c}\right)^n$

   (B) $c\left(\frac{n}{c}\right)$

   (C) $(cn)^n$

   (D) $n(cn)$

P.T.O.



2. In dynamic programming problems, the rule which determines the decision at each stage is known as:

   (A) State

   (B) Decision

   (C) Stage

   (D) Policy

3. In dynamic programming problems, the main problem is divided into subproblems. Each sub-problem is known as:

   (A) Stage

   (B) Decision

   (C) State

   (D) None of the above

4. If the outcome at any decision stage is unique and known for the problem, then the dynamic programming problem is known as :

   (A) Probabilistic dynamic programming problem

   (B) Stochastic dynamic programming problem

   (C) Static dynamic programming problem

   (D) Deterministic dynamic programming problem

5. If the value of the game is zero, then the game is known as :

   (A) Fair strategy

   (B) Pure strategy

   (C) Pure game

   (D) Mixed game

6. In case there is no saddle point in a game then the game is :

(A) Deterministic game

(B) Fair game

(C) Mixed strategy game

(D) None of the above

7. If the losses of player *A* are the gains of the player *B*, then the game is known as :

(A) Fair game

(B) Unfair game

(C) Nonzero sum game

(D) Zero sum game

8. The value of the game for the following payoff matrix is :

$$\begin{array}{cc} & A \\ B & \begin{bmatrix} 2 & 3 \\ -5 & 5 \end{bmatrix} \end{array}$$

(A) 5

(B) 3

(C) 2

(D) -5





9. In an integer programming,

(A) All coefficients of objective function must be integer

(B) All right-hand side values of constraints must be integer

(C) All variables must be integer

(D) All of the above

10. Indentify the statement which is correct in the context of mixed integer programming problems:

(A) The decision variables can take integer values only but the slack/surplus variables can take fractional values as well.

(B) The decision variables can take fractional values only but the slack/surplus variables can take integer values as well.

(C) Only few of the decision variable requires integer values.

(D) Only few of the slack/surplus variables requires integer values.

11. Consider the following statements :

(I) A travelling salesman problem can be solved using branch and bound method.

(II) An integer programming problem that has no constraints is known as Knapsack problem.

(A) (I) is true and (II) is false

(B) (I) is false and (II) is true

(C) (I) and (II) both are false

(D) (I) and (II) both are true

12. Indentify the statement which is not correct in the context of branch and bound method :

    (A) It divides the feasible region into smaller parts by the process of branching.

    (B) It can be used to solve any kind of programming problem.

    (C) It is not a particular method and is used differently in different kinds of problem.

    (D) It terminates when the upper and lower bounds become identical and the solution is the single valueable.

13. In $(M / M / 1):(\infty / FCFS)$ model, the length of the system $L_s$ is given by :

    (A) $\frac{P^2}{1-p}$

    (B) $\frac{P}{1-p}$

    (C) $\frac{\lambda^2}{\mu-\lambda}$

    (D) $\frac{\lambda^2}{\mu(\mu-\lambda)}$





14. For a simple queue $(M / M / 1), p=\frac{\lambda}{\mu}$ is known as :

    (A) Poisson busy period

    (B) Random factor

    (C) Traffic intensity

    (D) Exponential service factor

15. When the operating characteristics of the queue system dependent on time, then it is said to be :

    (A) Steady state

    (B) Explosive state

    (C) Transient state

    (D) None of the above

16. The unit of traffic intensity is :

    (A) Poisson

    (B) Markow

    (C) Erlang

    (D) None of the above

17. In the context of the general non-linear programming problem with equality constraints.

    (A) A non-linear programming probloem is solved using Lagrange multipliers

    (B) The sufficient conditions can be verified by the bordered Hessian matrix

    (C) Both (A) and (B)

    (D) None of the above

18. In the context of the general non-linear programming problem,

    (A) The constraints may be of the $\leq, = or \geq type$

    (B) The concept of partial derivatives is used to optimize the multivariable function

    (C) If the objective function is strictly concave, the Kuhn-Tucker conditions are sufficient conditions for an absolute maximum

    (D) All of the above

19. The quadratic form $x^T Qx$ is said to be negative-definite, if

    (A) $x^T Qx > 0$

    (B) $x^T Qx < 0$

    (C) $x^T Qx \geq 0$

    (D) $x^T Qx \leq 0$





20. Quadratic programming is concerned with the non-linear programming problem of optimizing the quadratic objective function subject to -

    (A) Linear inequality constraints

    (B) Non-linear inequality constraints

    (C) Linear equality constraints

    (D) Non-linear equality constraints

**(Section-B)**

**(Very Short Answer Type Questions)**

**(2 marks each)**

**Note: Attempt all questions:**

1. Define dynamic programming problem.
2. What is non-zero sum game?
3. Define maximin-minimax principle in game theory.
4. Define integer programming problem.
5. What is queueing theory?
6. Define steady state in queueing system.
7. Define quadratic programming problem.
8. Define convex programming problem.

## (Section-C)

## (Short Answer Type Questions)

**(3 marks each)**

**Note: Attempt all questions:**

1. Write the steps of dynamic programming algorithm.
2. Write the dominance property in game theory.
3. In a game of matching coins with two players, suppose *A* wins one unit of value, when there are two heads, wins nothing when there are two tails and loss ½ unit of value when there are one head and one tail. Determine the payoff matrix, the best strategies for each player and the value of the game to *A*.
4. Write the steps of Gomory's all integer cutting plane method.
5. Explain Kendall's notations for representing queueing models.
6. A supermarket has a single cashier. During the peak hours, customers arrive at a rate of 20 per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate:

   (i) The probability that the cashier is idle.

   (ii) The average number of customers in the queueing system.

   (iii) The average time a customer spend in the system.







7. Obtain the set of necessary conditions for the following non-linear programming problem:

   Minimize

   $z = 2x_1^2 + 2x_2^2 + 2x_3^2 - 24x_1 - 8x_2 - 12x_3 + 200$

   Subject to the constraints :

   $$x_1 + x_2 + 3x_3 = 11$$
   $$5x_1 + 2x_2 + x_3 = 5$$

8. Write the steps of separable programming algorithm.

## (Section-D)

## (Long Answer Type Questions)

**(4 marks each)**

**Note: Attempt all question:**

1. Use dynamic programming to solve the following program.

   Minimum

   $$z = x_1^2 + 2x_2^2 + 4x_3$$

   Subject to the constraints :

   $$x_1 + 2x_2 + x_3 \geq 8,$$
   $$x_1, x_2 x_3 \geq 0.$$

   **OR**

   Use dynamic programming to solve the following L.P.P

   Maximize

   $$z = x_1 + 9x_2$$

   Subject to the constraints :

   $$2x_1 + x_2 \leq 25,$$
   $$x_2 \leq 11,$$
   $$x_{1,} x_2 \geq 0.$$

2. Two companies *A* and *B* are competing for the same product. Their different strategies are given in the following payoff matrix :

$$\begin{array}{cc} & \text{Company}A \\ \text{Company}B & \begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix} \end{array}$$

Use linear programming method to determine the best strategies for both the players.

**OR**

Solve the game whose payoff matrix is given below by graphical method :

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A & \begin{bmatrix} 4- & 2 & 3 & -1 \\ -1 & 2 & 0 & 1 \\ -2 & 1 & -2 & 0 \end{bmatrix} \end{array}$$

3. Solve the following mixed integer programming problem: Maximize

$$z = x_1 + x_2$$

Subject to the constraints :

$$2x_1 + 3x_2 \leq 5,$$
$$x_1 + 2x_2 \leq 10,$$
$$x_{1,}x_2 \geq 0 \, and . x_1 is\ an\ integer$$

**OR**

Use branch and bound method to solve the following L.P.P.
Maximize

$$z = 7x_1 + 9x_2$$

Subject to the constraints :





$$-x_1 + 3x_2 \leq 6,$$
$$7x_2 + x_2 \leq 35,$$
$$x_{1,}x_2 \geq 0 \qquad and\ \ are\ \text{int}\,eger.$$

4. If the arrivals are completely random, then show that the probability distribution of numbers of arrivals in a fixed time interval follows a Poisson distribution.

**OR**

Explain $\{(M / M / 1) : (\infty / FCFS)\}$ system and solve it under steady-state conditions.

5. Use Wolfe's method to solve the following quadratic programming problem :
Maximize

$$z = x_1^2 - x_1 x_2 + 2x_2^2 - x_1 - x_2$$

Subject to the constraints :

$$2x_1 + x_2 \leq 1,$$
$$x_1, x_2 \geq 0.$$

**OR**

Use Beale's method to solve the following non-linear programming problem :
Maximize

$$f(x) = 2x_1 + 3x_2 - x_1^2$$

Subject to the constraints :

$$x_1 + 2x_2 \leq 4,$$
$$x_1, x_2 \geq 0.$$